Management of Intellectual Asset Production in Industrial Laboratories

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Abstract: Industrial laboratories generate profit for their parent companies and in so doing benefit society through spillovers of novel technologies and solutions. Yet research's share of corporate investment in R&D has been declining. To understand this trend from the operations perspective, we develop a model-based analysis of the management of intellectual asset production in industrial laboratories. The model consists of a linear network with multiple stages in which the first stage is the research division engaged in generating novel concepts and prototypes. It is followed by multiple development stages that transform research outputs into intellectual assets and marketable products. Management is responsible for strategic budget allocation to the stages, and tactical management of individual projects. Decisions are based on intrinsic return on investment in the laboratory, and option values of projects, both of which are endogenously determined. Our model and analyses have revealed several possible pathways that can lead to the management of the laboratories to reduce the share of research spending in their budgets, namely, a) lower variability of project values; b) improved investment efficiency at development stages; and c) higher revenue realization from assets produced at early development stages.

Keyword: Research and Development, Operations, Intellectual Assets, Innovation, Industrial Laboratories.

1 Introduction

Corporate R&D is an engine of technology innovations. In contrast to "open science" [6], which describes public-funded research conducted in universities and government laboratories, corporate researchers conduct "commercial science", where the mission is to generate profits by creating, developing, and harvesting intellectual assets. These assets are incorporated in novel products, solutions, and patents.

Society benefits, directly from the consumption and exercise of products and solutions, and the use through patent licensing of knowledge that is created, and indirectly from the spillover effect. Nevertheless, studies of American corporations' commitment to research and development have shown that investment in R&D has flatlined over the last several years as a share of the economy at about 2.9% of the nation's GDP. There is also strong evidence that the distribution is changing significantly away from research to development [1], [2]. This development has many societal implications, such as the loss of opportunity for enhancing peoples' well-being and the decay of national competitiveness in a globalized economy. Understanding some of the underlying forces that drive this trend is an essential prerequisite for policy-makers to address it effectively.

Motivations and Goals: Economists have pointed to the many changes in the macroeconomic environment and industry organization as reasons for reduced corporate funding for research. Various explanations have been proposed and tested in empirical studies. These explanations are generally supported by established economic theory, including some going back to the work of Schumpeter [24], and they present a picture of incentives and institutions at the national, industry, and corporation levels (see e.g., [23], p.11). However, to better understand this trend, it is also necessary to complement macroanalysis with the perspective of R&D managers who decide where to put resources and what projects to work on. Managers make these operational decisions to maximize profit, in the confines of their micro-environments. As the environments change, so do the choices. A main purpose of this paper is explore possible changes of environment that may cause reductions in research spending.

There is a considerable amount of prior work on model-based analysis of industrial R&D. Many studies focus on the optimal management of R&D projects, giving rise to not only various model formulations and solutions, but also new analytical frameworks and managerial insights (see, e.g., [7], [14], [13] [15], [16], [17], [25]). Notably, the idea of a correspondence between project executions and financial options is widely discussed. For instance, Smith and Nau reveal the connection between the option-price based analysis and decision-tree based analysis of project management decisions [25]. Hechzermeier and Loch consider different types of project uncertainties and assess the value of various options of handling them. They show that the value of having these options may increase or decrease as the project uncertainties increases, depending on the source of the uncertainty [15]. Their study is expanded by Kettunen et. al, which carries out the analysis against the backdrop of a competitive market environment that evolves over time [16]. Also, the optimization model conceived and analyzed by Hall et. al addresses not only randomness of returns but also uncertainty in distributions of returns [13].

Our study focuses on industrial laboratories because they are major institutional sources of innovations. As Arrow points out [3], inventive activities are riddled with economic conflicts caused by the nature of the new information generated and uncertainties about their values. An insurance mechanism that encourages firms to take risk can also shield them from consequences of poor decisions, and thus depress their incentives to be successful. He argues that "The only way, within the private enterprise system, to minimize this problem is the conduct of research by large corporations with many projects going on, each small in scale compared with the net revenue of the corporation"[3]. Industrial laboratories exemplify the institution and the process.

Specifically, we develop a model of industrial laboratories and analyze and optimize its primary functions, which are the generation, development, and harvesting of intellectual assets. Our model consists of a research stage, which generates new concepts and prototypes, followed by processing stages that selectively develop some of these projects. The output of the research stage is not the result of planned decisions, but rather a stream of "projects", with each project originating at an uncertain time and of uncertain value initially, but having the potential to be developed into a profitable product. Projects are screened and gated at each stage. Profit is generated not only from projects that reach the terminal marketable state, but also through licensing of incremental advances at preceding stages.

We use the model as a platform to carry out "logic experiments", by which we mean that by varying model parameters and decisions, especially in budget allocations to research and development, we develop an understanding of their impact on the return on investment in the industrial laboratory and, in particular, possible reasons for reduced spending in research.

Our Approach: In comparison to aforementioned studies, our formulation takes a broader approach. A new and essential feature of our model of the industrial laboratory is the structural and functional separation among the various stages. They are interdependent, as is implied, for instance, in the linear network topology, but each stage has a distinct role. The research stage, however, is special since it is in a sense the engine of the laboratory. In particular, to understand the decline in research spending it is necessary to have such a model with structural and functional details.

Project management is an essential part of our model. We consider flows of projects instead of an individual project as the object of our study. Instead of differentiating various types of uncertainties in the R&D process, we capture uncertainty in markets and payoffs, and its implications in Markov chains and their limiting approximations. Also, instead of considering many controls in managing projects, we focus on the most important one, the decision to continue or abandon certain classes of projects.

While simplifying some project management decisions, we also extend our model formulation along other dimensions to serve the purpose of our study. In particular, we integrate project generation and development functions into a single model and combine budget allocation and project management as a joint optimization problem. More notably, we conceive the concept of "asset banks", which refer to collections of patents and other other value-generating intellectual properties. Each bank is associated with research or a development stage, and is replenished by the activities at the stage (research) for generating new projects, or for their transformation to products (developments).

As illustration of the importance of asset banks is the observation by Gerstner [9] that during IBM's turnaround during his tenure as CEO, income from licensing, patents, royalties and sale of intellectual properties grew to $1.5 billion in 2004. Another reference point is that in 2012, Alcatel-Lucent, which owned Bell Labs, used its portfolio of 30,000 patents valued at $6.5 billion as collateral to secure a loan of $2.1 billion from Credit Suisse and Goldman Sachs [5]. In both cases, the laboratories possessed R&D products in their asset banks that generated financial gains for the parent companies, which were separate from the contributions of the laboratories in the companies' sales of final products.

Findings: We develop key performance metrics, such as the intrinsic return on investment in the laboratory, and option values of projects, that unify budget allocation and project management decisions. By analyzing these metrics and conducting numerical experiments, we have found the following plausible causes for the decline of research spending in the industrial laboratories.

Variability of Project Values: It is well known that corporate R &D benefits from high variability in project values. In this case, there are potentially large gains to be obtained by developing high-value projects, while downside losses can still be contained by terminating least promising ones [8]. Our model yields a similar outcome: as the differences in project values grow, the return on investment increases.

More important than confirming existing conclusions, our analysis also reveals the impact of the variability on research spending. We find that when there is sufficient differentiation of project values, the optimal strategy is to invest heavily in research to create a large pool of projects. The management can then apply its selection function to aggressively eliminate weak projects, and rely on a few high-value projects for profit generation. As the difference diminishes, this strategy ceases to be optimal and the fraction of budget allocated to research decreases, i.e., the investment becomes increasingly more focused on completing existing projects than generating new ones.

This finding implies a possible reason for less investment in research, namely, the output of the research stage shifts from a combination of many failed projects and a few truly ground-breaking innovations to a stream of "safe" projects with similar values. This can happen for any of several possible reasons, such as the lack of significant technological advances, or breakthroughs, or market evolutions, or managers' risk-averseness.

Cost-Efficiency of Project Development: Empirical studies have shown that companies spend more on research if they can make more successful internal use of its results [2]. Extending this argument to the management of industrial laboratories, one may expect that there will be more investment in research if it is less costly to develop resulting new ideas and projects into marketable products.

However, this expectation is not consistent with the observation that the current decline in research spending is happening in an environment where the cost-efficiency of development is rapidly improving. For instance, the Internet enhances developers' ability to learn, communicate, and collaborate; high-fidelity simulations provide cheaper alternatives to physical experiments and field tests; 3-D printing makes rapid and inexpensive prototyping possible; and open-source computer codes allow the laboratories to avoid costly development of software from the scratch.

Our study shows that there are more nuances in the influence of cost-efficiency of development on the research spending. In numerical experiments with our model, we consider cases in which there is decrease in the required investment to increment an unit of development capacity. We also consider cases in which the bulk of investment needed for development occurs in the latter stage, so there will be less waste on projects that are abandoned at early stages. In both cases, we find a cyclic pattern of changes in investment in research as the cost efficiency of development improves. There are some continuous increases in the share of research spending because the improvements frees up funds. However, the trend is interrupted at many interim points where the share of research spending drops significantly. This is because more efficient development can also make it profitable to not abandon projects with lower values, which means investment in research has to be diverted to support additional development efforts.

Early Realization of Project Values: We show that the optimal budget allocation in industrial laboratories should adapt to the revenue generated at research and different development stages. For instance, the optimal solution to our model shows that when the revenue generated from licensing intellectual property originating in the research stage dominates revenue from developed products, then the laboratory may give up development to become a pure "knowledge factory". Similarly, if more revenue is generated at upstream development stages, then it may be beneficial to shut down the late development stages and forgo the launch of final products.

In fact, industrial laboratories do quit developing final products when competition makes them less profitable. Instead, efforts are concentrated on early-stage development to capitalize research outcomes through patents, tools, and consulting services. Surprisingly, the reduction of the scope of development often leads to less spending on research, and the decreases is often attributed to the overall reduction of the laboratories' budget.

Our study shows that R&D budget cuts are not the only explanation. With the total budget fixed, our numerical experiments show that as the fraction of the total revenue generated at the early development stages increases, it can be optimal to both close late development stages and also shift the budget allocated to research to development. This is because when profit can be quickly realized, the absence of uncertainty and the cost involved in further development of projects into final products, it is profit-maximizingl to be less rigorous with project selection at early development stages, which increases the development load and drains investment away from research.

In the rest of the paper, we define our model in Section 2, present a partial (fluid-scale) optimal solution and discuss implications for management in Section 3. In Section 4 we specialize the model to just two stages, and obtain explicit solutions that support further managerial insights. We report on numerical case studies in Section 5 and conclude in Section 6.

2 Model

Our model for industrial laboratories is built from three basic elements: a multi-stage system representation of the end-to-end research and development processes; an information discovery process that progressively reveals the potential of projects to generate revenue and assets, and a set of asset banks that contain assets such as patents and other intellectual properties that generate revenues during their finite economic lives.

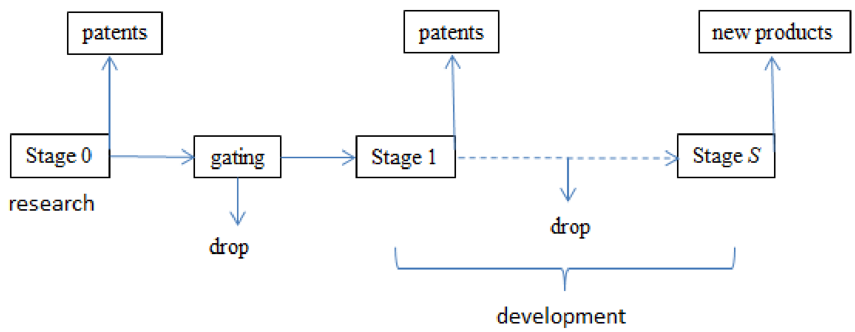

Figure 1: A Model of Industrial Laboratory

The multi-stage system is shown in Figure 1. The initial stage (stage 0) is where the open-ended research is conducted to create novel concepts and prototypes. Each entity is the starting point of a potential project. A project is transformed, implemented, and tested and possibly terminated at subsequent development stages s ($s = 1, ..., S$). The latter stages are staffed by development teams with specialized skills. When a project reaches the final stage S, the research output has been developed into a marketable product.

Asset banks exist in each stage. An asset bank is a pool of intellectual assets that generate revenue over time, e.g., patents that can be licensed for fees or traded for other patents to avoid paying their holders. The asset bank at the last stage also includes new products, which generate gross profits, i.e., revenue minus variable costs of making and selling these products, for the company. To be concise, we will continue to refer to these gross profits are revenues for the laboratory, as the latter variable cost are incurred by outside business units.

As a project progresses along stages, its value is determined by the revenue that it generates in the current stage, and, if the project continues to the next stage, the revenue it is expected to generate in subsequent stages, minus the associated development costs. The overall value is not known at the start but is increasingly ascertained and incrementally realized as the project moves through the development stages. After each stage, a project goes through an assessment, which is used to determine whether it should be forwarded to the next stage for further development or terminated. We call this the gating process, see Figure 1.

An example of a multi-stage R&D process is the fundamental research on neurobiology and psychoacoustics in Bell Labs in the late 1970s, which revealed the role of frequency resolution and masking, which in turn, spawned the development of synthetic speech and novel CODECs for improving voice quality on telephone networks [18]. Development of the project also spun-off vast amounts of intellectual assets, such as patented compression algorithms and, notably, the MPEG standards. The value of these assets are in billions of dollars, but many of them will expire and be replaced by newer and better algorithms. Our model captures this process in its entirety via asset banks that hold revenue-generating assets, which are created at different stages and have finite lifetimes. In some industries, such as pharmaceutical laboratories, it is often the case that revenues, even those from patents, can be derived only after completion of the end-to-end development ([10], [22]). Such cases are also covered as special cases of our model.

2.1 Markov Model of System

Uncertainty is inherent to the R&D process. To handle uncertainty we introduce a Markov model of the industrial laboratory, and define its key variables and parameters.

To model differences in revenue generation between projects, we define, for each stage, a set of states that are specific to it, and their transition probabilities. Projects belonging to the same state produces assets that generate similar amounts of revenues at the current stage and are also expected to do the same as they continue developments at later stages. Due to randomness, not all projects that start from the same state at a stage will evolve into the same state after processing at the next stage. On the other hand, it is reasonable to expect that the current state of a project should have an important role in determining its state at the next stage. For these reasons, we denote the set of project states at stage s by $\mathcal{K}_s$ $(s = 0, 1...., S)$, define q^0_k to be the transition probability that a research output is in state $k \in \mathcal{K}_0$ and q^s_{jk} to be the probability that an incoming project in state

j $(j \in \mathcal{K}_{s-1})$ will be in state k $(k \in \mathcal{K}_s)$ after its development at stage s $(s = 1, ..., S)$.

Let $\tilde{\Theta}_0$ be the random variable representing the number of research projects generated in stage 0 per unit of time. Let $\Theta_{0,k}$ be the random variable representing the number of these projects that are assessed to be in state k in $\mathcal{K}_0$. Then

$$\Theta_{0k} = \mathcal{M}_k(\tilde{\Theta}_0, \mathbf{q}^0), \text{ where } \mathbf{q}^0 := \left(q_1^0, ..., q_{|\mathcal{K}_0|}^0\right), \ k \in \mathcal{K}_0, \tag{1}$$

where $\mathcal{M}_k(\tilde{\Theta}, \mathbf{q})$ is the number of realizations of outcome k of a multinomial distribution with $\tilde{\Theta}$ as the number of trials and $\mathbf{q}$ as the probability vector. For each unit of time, let Λ_{sj} be the random number of projects that make the transition from state j $(j \in \mathcal{K}_{s-1})$ to the development stage s $(s = 1, ..., S)$ in unit time, and Θ_{sk} be the random number of projects that complete development at stage s $(s = 1, .., S)$ in state $k \in \mathcal{K}_s$ in unit time. Hence, the total number of projects admitted to stage s in unit time is

$$\tilde{\Lambda}_s = \sum_{j \in \mathcal{K}_{s-1}} \Lambda_{sj},$$

and the number of processed projects in unit time in state k, and over all states are, respectively,

$$\begin{aligned} \Theta_{sk} &= \sum_{j \in K_{s-1}} \mathcal{M}_k(\Lambda_{sj}, \mathbf{q}_j^s), \text{ where } \mathbf{q}_j^s := \left(q_{j1}^s, ..., q_{j|\mathcal{K}_{s-1}|}^s\right), j \in \mathcal{K}_{s-1}, k \in \mathcal{K}_s, s = 1, \cdots, S, \\ \tilde{\Theta}_s &= \sum_{k \in \mathcal{K}_s} \Theta_{sk}, \quad s = 1, \cdots, S. \end{aligned} \tag{2}$$

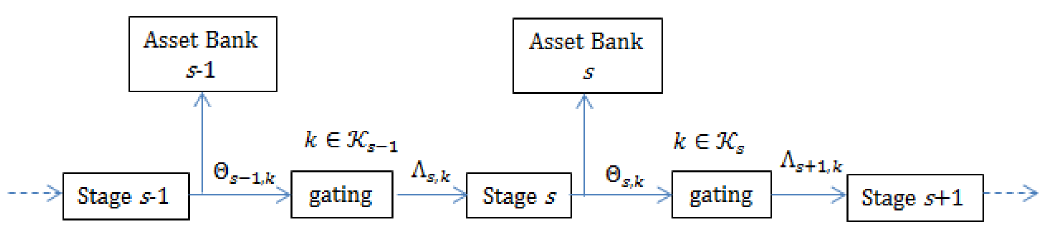

Figure 2: Variables at Stage s

Turning to the asset banks, the rate of revenue generated by an asset per unit of time, r_{sk}, depends on both the stage s $(s = 0, 1, \cdots, S)$ and state $k \in \mathcal{K}_s$. Depending on the stage, r_{sk} may be viewed as the average licensing fee per unit of time per patent, or the average gross profit (i.e., revenue minus variable cost) of a new product collected per unit of time. This average rate can be zero if the corresponding bank holds worthless inventions or obsolete products.

Intellectual assets are subject to obsolescence and consequent depreciation of their value over time. Hence each bank is associated with a birth-death process, where the "birth" corresponds to production and "death" with the end of the revenue-generating lifetime of the assets. For each bank, we assume that the birth rate of its assets is proportional to the output rate of projects of the corresponding state, and, without loss of generality, let the constant of proportionality be unity. The death rate is inversely proportional to the average economic lifetimes of assets and denoted by μ_{sk} ($k \in \mathcal{K}_s$, $s = 0, 1, ..., S$).

This concludes the description of the Markov model. Below we proceed with model development by specializing in two steps. In the first, the Markovian system is assumed to be in its stationary state, i.e., in steady state, and the quantities of interest are expected, i.e., mean, values. In the second, the steady state mean values are further specialized to their deterministic counterparts obtained from a fluid-scale approximation. These specializations are respectively undertaken in Sec. 2.2 and 3.1.

2.2 Stationary Markov System and Mean Values

We now assume that the Markov system is in equilibrium and define mean values of quantities of interest for the stationary system. Define

$$\lambda_{sk} = E[\Lambda_{sk}] \ (k \in \mathcal{K}_{s-1}) \quad \text{and} \quad \theta_{sk} = E[\Theta_{sk}] \ (k \in \mathcal{K}_s)$$

These mean values have natural interpretations as rates, and we shall find it convenient to refer to them as such. Thus, λ_{sk} and θ_{sk} are respectively input and output rates of projects in state k at stage s. Similarly,

$$\tilde{\lambda}_s = E[\tilde{\Lambda}_s] \quad \text{and} \quad \tilde{\theta}_s = E[\tilde{\Theta}_s]$$

are the total rates for stage s. The output rates for the research stage, θ_{0k} $(k \in \mathcal{K}_0)$ and $\tilde{\theta}_0$ are defined similarly.

Analogous to the transitions in the Markov model, we obtain $\theta_{sk} = \sum_{j \in \mathcal{K}_{s-1}} q^s_{jk} \lambda_{sj}$ $(k \in \mathcal{K}_s$, $s = 1, ..., S)$. At the research stage, $\theta_{0k} = q^0_k \theta_0$, $k \in \mathcal{K}_0$.

In the gating process the manager selects only certain types of projects that have completed development in stage $s-1$ for further development in stage s. This process imposes the constraint

$$0 \le \lambda_{sk} \le \theta_{s-1k}, \quad k \in \mathcal{K}_{s-1}, \; s = 1, ..., S,$$

since the input rate to stage s in state k cannot exceed the output rate at stage $s-1$ in state k. The slack in the second inequality reflects the effects of gating between stages $s-1$ and s.

We next consider the mean values of the asset banks. Let $\mathcal{B}_{sk}$ be the size of asset bank in steady state $(k \in \mathcal{K}_s, \; s = 0, ..., S)$. Recall that the evolution of an asset bank is described by a birth and death process in which the birth rate is θ_{sk} and the death rate is μ_{sk} for $(k \in \mathcal{K}_s, s = 1, \cdots, S)$. Hence the expected size of the asset bank at steady state

$$B_{sk} := E[\mathcal{B}_{sk}] = \theta_{sk}/\mu_{sk}, \quad k \in \mathcal{K}_s, \; s = 0, ..., S. \tag{3}$$

Also r_{sk} is the amount of revenue generated by an asset per unit of time and $1/\mu_{sk}$ is the expected economic life of an asset. Thus

$$w_{sk} := r_{sk}/\mu_{sk}, \quad k \in \mathcal{K}_s, s = 0, \cdots, S \tag{4}$$

is the expected amount of revenue generated by an asset over its lifetime. Given the birth rate θ_{sk}, the rate of revenue generation is

$$w_{sk}\theta_{sk} = r_{sk}B_{sk} = (r_{sk}\theta_{sk})/\mu_{sk}, \; k \in \mathcal{K}_s, \; s = 0, \cdots, S. \tag{5}$$

2.3 Allocations, Costs and Return on Investment

We shall assume that the parent corporation invests the amount I in the laboratory, which the manager is authorized to spend. The laboratory manager's responsibilities include allocation of funds for spending in each stage, and project gating. The allocation to the research stage is denoted I_0. While the actual volume of research output generated per unit of time is stochastic, the mean rate is a deterministic increasing function of I_0, which we denote by $C_0(I_0)$. Spending in stage s is denoted by I_s, which determines the processing capacity, C_s $(s = 1, ..., S)$. The total spending on all stages cannot exceed the corporate investment, I, i.e.,

$$I_0 + ... + I_S \leq I. \tag{6}$$

The exact form of $C_s(I_s)$ $(s = 1, \cdots, S)$ depends on the nature of the development. For instance, if the development corresponds to clinical trials, then higher spending allows more trials to be conducted simultaneously but the duration of each trial cannot be shortened. Thus capacity increase is due to the increased number of projects that can be simultaneously developed, not the shorter development span of each project. On the other hand, in the case of software development, higher spending translates to a larger number of teams of more experienced developers, so that higher capacity is due to the increased number of concurrent projects that can be developed and a potentially higher rate of developed projects. In many cases a development stage s $(s = 1, \cdots, S)$ can function only after its capacity reaches a critical mass, which means that $C_s(I_s) > 0$ only after I_s exceeds a threshold. The above considerations can be captured by specific forms of $C_s(I_s)$, and the complexity of the model and its solution vary with the formulation. To focus on our main message, we will assume a simple form that reflects the common first-order effect.

We assume that the development of projects also incurs a direct cost per project, c_{sk} $(k \in \mathcal{K}_{s-1}$, $s = 1, ..., S)$. Then the mean net return from the investment in the laboratory per unit of time is

$$\sum_{s=0}^{S} \sum_{k \in \mathcal{K}_s} w_{sk}\theta_{sk} - \sum_{s=1}^{S} \sum_{k \in \mathcal{K}_{s-1}} c_{sk}\lambda_{sk}. \tag{7}$$

3 Fluid-Scale Model and Optimization

In this section, we first give the system of equations that describe the deterministic approximation of the mean values of the stationary Markov model obtained above. Next we obtain the solution and discuss its use to support management. Proposition 1 below derives a set of unified metrics for assessing the financial performance of the laboratory in its entirety and the value of individual R&D projects. Proposition 2 articulates how these metrics may be applied in making investment and project selection decisions. More than a prescription for action, these results give direction to a strategy for sustaining and growing the laboratory.

3.1 Fluid-Scale Model

The fluid-scale approximation treats the mean values, i.e., rates, of quantities of interest in the stationary Markov model as deterministic quantities. Specifically, the quantities of interest, which have been defined in Sec. 2.2 are λ_{sk}, θ_{sk}, $\tilde{\lambda}_s$, and $\tilde{\theta}_s$ ($k \in \mathcal{K}_s$, $s = 0, \cdots, S$). For instance, here we interpret λ_{sk} and θ_{sk} to be the deterministic input and output rates of projects of state k at stage s ($k \in \mathcal{K}_s, s = 0, \cdots, S$).

In analogy to the Markovian formulation, $\theta_{sk} = \sum_{j \in \mathcal{K}_{s-1}} q^s_{jk} \lambda_{sj}$ ($k \in \mathcal{K}_s$, $s = 1, ..., S$). At the initial, research stage, $\theta_{0k} = q^0_k \theta_0$ ($k \in \mathcal{K}_0$). We assume the particularly simple linear relationship between the processing capacity and investment at each stage s, i.e.,

$$I_s = \gamma_s C_s, \; 1 \leq s \leq S.$$

We refer to the parameter γ_s as the (required) "investment per unit of development capacity" (increment) in stage s ($s = 1, \cdots, S$). Similarly, the research output rate θ_0 is assumed to be linearly related to the investment in research, $I_0 = \gamma_0 \theta_0$.

3.2 Fluid-Scale Optimization

With these specifications, the fluid-scale optimization is formulated as the following LP:

$$\max_{\underline{\lambda}, \underline{\theta}, \mathbf{C} \geq 0} \left\{ \sum_{s=0}^{S} \sum_{k \in \mathcal{K}_s} w_{sk} \theta_{sk} - \sum_{s=1}^{S} \sum_{k \in \mathcal{K}_{s-1}} c_{sk} \lambda_{sk} \right\} \tag{8}$$

$$\text{s. t.} \quad \gamma_0 \theta_0 + \gamma_1 C_1 + ... + \gamma_S C_S \leq I, \tag{9}$$

$$\lambda_{s+1k} \leq \theta_{sk}, \quad k \in \mathcal{K}_s, \; s = 0, ..., S-1, \tag{10}$$

$$\theta_{sk} = \sum_{j \in \mathcal{K}_{s-1}} q^s_{jk} \lambda_{sj}, \quad k \in \mathcal{K}_s, s = 1, ..., S, \tag{11}$$

$$\sum_{k \in \mathcal{K}_{s-1}} \lambda_{sk} \leq C_s, \quad s = 1, ..., S, \tag{12}$$

$$\theta_{0k} = q_k^0 \theta_0, \quad k \in \mathcal{K}_0. \tag{13}$$

Using (11) in the first term in (8),

$$\sum_{k \in \mathcal{K}_s} w_{sk}\theta_{sk} = \sum_{k \in \mathcal{K}_s} w_{sk} \sum_{j \in \mathcal{K}_{s-1}} q_{jk}^s \lambda_{sj} = \sum_{j \in \mathcal{K}_{s-1}} \tilde{w}_{sj}\lambda_{sj}$$

where $\tilde{w}_{sj} := \sum_{k \in \mathcal{K}_s} w_{sk} q_{jk}^s$ $(j \in \mathcal{K}_{s-1},\ s = 1, ..., S)$ may be viewed as the expected asset value of incoming projects in state j after processing at a single stage s, i.e., without taking into account changes in subsequent stages. Without loss of generality, we normalize γ_0 to 1 and transform the above optimization problem into the following:

$$\max_{\underline{\lambda}, \theta_0, \mathbf{C} \geq 0} \left\{ \theta_0 \sum_{k \in \mathcal{K}_0} w_{0k} q_k^0 + \sum_{s=1}^{S} \sum_{k \in \mathcal{K}_{s-1}} (\tilde{w}_{sk} - c_{sk})\lambda_{sk} \right\} \tag{14}$$

$$\text{s. t.} \qquad \theta_0 + \gamma_1 C_1 + ... + \gamma_S C_S \leq I, \tag{15}$$

$$\lambda_{1k} \leq q_k^0 \theta_0 \quad (k \in \mathcal{K}_0), \tag{16}$$

$$\lambda_{sk} \leq \sum_{j \in \mathcal{K}_{s-2}} q_{jk}^{s-1} \lambda_{s-1j} \quad k \in \mathcal{K}_{s-1}, s = 2, ..., S, \tag{17}$$

$$\sum_{k \in \mathcal{K}_{s-1}} \lambda_{sk} \leq C_s, \ \ s = 1, ..., S. \tag{18}$$

The insights from the optimization of the fluid model are more forthcoming by examining the dual version of the LP in (14)-(18). Let α, β_{1k} $(k \in \mathcal{K}_0)$, β_{sk} $(k \in \mathcal{K}_{s-1},\ s = 2, ..., S)$, and η_s $(s = 1, ..., S)$ be the dual variables associated with constraints (15), (16), (17), and (18), respectively. We interpret α as the intrinsic return on investment in the laboratory; η_s as the shadow price of processing capacity of stage s $(s = 1, ...S)$; and β_{sk} as the option value of a project in state k $(k \in \mathcal{K}_s)$ after its development at stage $s - 1$ $(s = 1, .., S)$, i.e., the value of not abandoning the project at stage s. The relationships of these quantities are given by the following proposition.

Proposition 1. *(see Appendix for the proof): The solution to the dual version in (14)-(18) of the Linear Programming problem is as follows:*

$$\alpha^* = \sum_{k \in \mathcal{K}_0} q_k^0 (w_{0k} + \beta_{1k}^*), \tag{19}$$

$$\eta_s^* = \alpha^* \gamma_s, \quad s = 1, ..., S, \tag{20}$$

$$\beta^*_{Sk} = (\tilde{w}_{Sk} - c_{Sk} - \alpha^*\gamma_S)^+, \quad k \in \mathcal{K}_{S-1}, \tag{21}$$

$$\beta^*_{sk} = (\tilde{w}_{sk} - c_{sk} + \sum_{j\in\mathcal{K}_s} q^s_{kj}\beta^*_{s+1j} - \alpha^*\gamma_s)^+, \;\; k \in \mathcal{K}_{s-1}, \;\; 1 \le s < S, \tag{22}$$

where α^* is a unique value.

The solution to (19)-(22) follows from the value of α^*. As is shown in (20), shadow prices η^*_s ($1 \le s < S$) are proportional to α^*. Option values β^*_{sk} ($k \in \mathcal{K}_{s-1}$, $1 \le s \le S$) can be determined by backward induction, using (21) for values at the last stage, and (22) for induction steps. Both equations imply that β^*_{1k} ($k \in \mathcal{K}_0$) decrease in α^*. Thus as solutions to (19), the latter values can be found by applying an iterative search procedure to the equation, with the aforementioned backward induction to compute option values.

3.3 Discussion of Results

Performance indicators are essential for effective management of the laboratory. When funding research and investing in development teams, managers need to understand how much bang they are getting for the buck. When deciding whether to advance or terminate a project in progress, they need to determine the potential profit, if any, that can be expected. Such assessments should not be made in isolation, but under a common standard that not only applies uniformly across different projects, but is also consistent across resources investments and project selections.

The proposition presents a system of performance metrics that may be used to satisfy this requirement. At the core is the intrinsic return on investment, α^*, which, as its name suggests, gives a holistic assessment of the overall value to be expected from each unit of investment in the laboratory. Multiplying α^* by the investment per unit of capacity (γ_s) gives the shadow price (η^*_s), which characterizes the per unit return on the investment in the development capacity at stage s ($s = 1, \cdots, S$). By this definition, η^*_s also serves as the implied cost (separate from the direct cost) for a project to be developed at stage s ($s = 1, \cdots, S$), accounting for the reduction of the availability of the capacity to other projects.

The value of α^* also underlies the appraisal of individual projects in different states. As is shown in (21), projects at the final stage are assessed by the net surplus of their asset value ($\tilde{w}_{Sk}$) to be generated over both the direct and implied development costs. When the value does not exceed the costs, the appraised value is set to zero, consistent with the manager's rational decision to terminate such projects. We refer to β^*_{sk} as the "option value" of projects in state k of stage s ($k \in \mathcal{K}_s$,$s = 1, \cdots, S$). As is shown in (22), the option values at stages preceding the final stage are determined recursively. The calculation includes not only their asset values and costs at the current stage, but also their expected option value at the next stage.

Thus our development in Proposition 1 defines an unifying basis for appraising the implied value/cost of development capacity and evaluating the potential worth of each project. Importantly, in our framework the value of α^* is determined endogenously for it to represent the actual return that can be generated from the laboratory. In (19), $\tilde{w}_{0k}$ is the asset value of an idea generated from research that can be harvested without further development, and β_{1k} is the option value of the idea that can be added from development. The sum of the two values yields the overall return, which is variable across states k ($k \in \mathcal{K}_0$). As (19) shows, the intrinsic return on investment is simply the sum of overall returns over all states.

To optimally channel value assessment to actionable decisions, the following three conditions need to be satisfied. First, projects at any given stage are advanced to the next stage if and only if they are associated with a strictly positive option value. Second, each development stage should be configured with enough capacity to handle all projects that pass the gating process, but not excessively so as to leave some of them idle. Finally, total investments in research and development should fit the given budget. The proposition below gives the solution that meet these requirements.

Proposition 2. *(see Appendix for the proof): Assume the non-trivial case where $\alpha^* > 0$. Then the following solution optimizes the primal fluid model,*

$$\theta^*_0 = I - \gamma_1 C^*_1 - .... - \gamma_S C^*_S, \tag{23}$$

$$C^*_s = \sum_{k \in \mathcal{K}_{s-1}} \lambda^*_{sk} \quad , s = 1, ..., S, \tag{24}$$

$$\lambda^*_{1k} = \mathbf{1}(\beta^*_{1k} > 0)\theta^*_0 q^0_k, \quad k \in \mathcal{K}_0, \tag{25}$$

$$\lambda^*_{sk} = \mathbf{1}(\beta^*_{sk} > 0) \sum_{j \in \mathcal{K}_{s-2}} q^{s-1}_{jk} \lambda^*_{s-1j}, \quad k \in \mathcal{K}_{s-1}, \ s = 2, ..., S. \tag{26}$$

A notable feature of the solution is its implied use of projects' option value to delineate the boundaries of the laboratory's research and development efforts. Consider a given stage s ($s < S$)

and a project in state k ($k \in \mathcal{K}_s$). If, in comparison with the intrinsic return on investment, α^*, the asset value that the project can generate beyond that stage is small, and/or the direct or implied cost of continuing developing the project is large, then the project's option value is zero. Following the solution in the proposition, $\lambda_{(s+1)k} = 0$, dictating the termination of the project at this point. If the situation applies to all projects at stage s, regardless of their states, then there will be no development at stages $s+1, \cdots, S$, and stage s becomes the end point of the laboratory's R&D.

Recall that α^* is endogenously determined from asset values and costs of all stages. Therefore, for the aforementioned situation to occur, it is not necessary that asset value of going beyond stage s is low, and the development cost of doing so is high *in their absolute values*. What is required is that in terms of value generated and cost incurred, the latter stages are at disadvantage comparing to earlier stages. In the presence of such imbalance, the laboratory should stop at stage s and have all its resources invested in its upstream stages even if it dominates alternative organizations in asset value and cost at every stage. This is the same rationale that justifies fabless manufacturing in the semiconductor industry.

The above results suggest a possible scenario in which the value of assets generated in the research stage dominates those generated at all development stages, and consequently the option value of developing any project (β^*_{sk}, $k \in \mathcal{K}_s$ and $s \geq 1$) becomes zero. To maximize return on investment in these cases, the laboratory should stop maintaining an end-to-end R&D process with its focus on the final product launch. Instead, it should evolve into a high-end "knowledge factory" that engages only in pre-development research and early incubation of concepts, and justify its existence by being a profit center with income derived solely from assets gained from the creation of new concepts and knowledge.

On the other hand, it is also possible that revenue generated by assets at development stages ($\tilde{w}_{sk}$, $k \in \mathcal{K}_s$, $s = 1, \cdots, S$) are high enough such that option values of projects β^*_{sk} are strictly positive in all states and at all development stages. An application of (25)-(26) is that it is optimal to develop all projects, in which case, the inequality constraints (16)-(17) of the primal LP become equalities. It may be verified that for given budget I, the investment per unit of development capacity $\boldsymbol{\gamma}$, and transition probabilities $\mathbf{q}$, the optimal value of θ_0 is the smallest in this case, i.e., industrial laboratories operating in this regime invest the least amount in research.

4 A System with Two Stages

In the simplest, yet non-trivial, case of our model, the laboratory has two stages, research and a single development stage. Also, projects incubated at the research stage belongs to either high or low value state (i.e., $\mathcal{K}_0 = \{h, l\}$), with probabilities $q_h^0 = q$ and $q_l^0 = 1 - q$ respectively. Let w_{0h} and w_{0l} denote the revenue generated by assets in the high and low states over their lifetimes, respectively. Then the expected revenue generated by a project at the research stage is $\tilde{w}_0 = q w_{0h} + (1 - q) w_{0l}$. We assume that the costs of developing both types of projects are the same, i.e., $c_{1h} = c_{1l} = c$. We denote the revenue that projects in states h and l are expected to generate after development by

$$\tilde{w}_h := \sum_{k \in \mathcal{K}_1} q_{hk}^1 w_{1k} \text{ and } \tilde{w}_l := \sum_{k \in \mathcal{K}_1} q_{lk}^1 w_{1k},$$

respectively. It is assumed without loss of generality that $\tilde{w}_h > \tilde{w}_l$.

We apply the solution from the fluid-scale optimization in Section 3 to this case. To simplify notations, we let β_h and β_l denote option values for developing projects in high and low value states, respectively, and λ_h and λ_l denote the corresponding rates of projects admitted for development. We let C be the capacity of the development stage, i.e., the maximum rate of projects that may be admitted for development, and γ be the investment required per unit of additional capacity. The optimal solution for the two-stage model is the following corollary to Propositions 1 and 2.

Corollary 1. *(see Appendix for the proof): It is optimal to invest only in research if and only if*

$$\tilde{w}_0 \geq (\tilde{w}_h - c)/\gamma, \tag{27}$$

in which case

$$\alpha^* = \tilde{w}_0, \quad \beta_h^* = \beta_l^* = 0, \quad \textit{and} \quad \theta_0^* = I, \quad \lambda_h^* = \lambda_l^* = C^* = 0. \tag{28}$$

If (27) does not hold, then the optimal solution depends on whether the following condition holds:

$$[\tilde{w}_0 + q(\tilde{w}_h - c)]/(1/\gamma + q) \geq \tilde{w}_l - c. \tag{29}$$

If the above inequality holds, then the optimal solution of the dual problem is

$$\beta_l^* = 0,\ \beta_h^* = (\tilde{w}_h - c - \gamma\tilde{w}_0)/(1+q\gamma),\ \alpha^* = (\tilde{w}_0 + q(\tilde{w}_h - c))/(1+q\gamma), \tag{30}$$

and that of the primal problem is

$$\lambda_l^* = 0,\ \lambda_h^* = C^* = qI/(1+q\gamma),\ \theta_0^* = I/(1+q\gamma). \tag{31}$$

If neither inequality in (27) nor (29) holds, then the optimal dual solution is

$$\begin{aligned}
\beta_l^* &= [(\tilde{w}_l - c) - q\gamma(\tilde{w}_h - \tilde{w}_l) - \gamma\tilde{w}_0]/(1+\gamma),\\
\beta_h^* &= [(\tilde{w}_h - c) + (1-q)\gamma(\tilde{w}_h - \tilde{w}_l) - \gamma\tilde{w}_0]/(1+\gamma),\\
\alpha^* &= (\tilde{w}_0 - c + q\tilde{w}_h + (1-q)\tilde{w}_l)/(1+\gamma),
\end{aligned} \tag{32}$$

and the solution to the primal problem is

$$\lambda_l^* = (1-q)I/(1+\gamma),\ \lambda_h^* = qI/(1+\gamma),\ \theta_0^* = C^* = 1/(1+\gamma). \tag{33}$$

Condition (27) specifies the special regime in which the intellectual assets generated in research are so valuable that it becomes more profitable to eliminate development and let the laboratory become a factory dedicated solely to research. In this case, optimality of investments gives an extreme form of imbalance in allocations.

In contrast, when the research stage does not generate sufficient revenue (i.e., $\tilde{w}_0$ is small so (27) does not hold), the laboratory should develop projects. The solution in (30)-(31) conveys an important managerial insight: for parameters in the region specified by (29), optimal investments are unbalanced but not nearly as extreme as in the previous case. Nevertheless, more projects are generated in the research stage than there is capacity to develop them. Projects in the low value state are aborted and the effort invested in them are lost, even though $\tilde{w}_l - c > 0$, i.e., the expected profit from development is positive.

This phenomenon reflects the different characteristics of research and development: the value of research output is unknown in advance, so for each project not generated, the laboratory loses an average value of a potential project. Once generated, assessments can be made to classify projects into different states. By first dropping projects in the low value state, the manager loses the below-average value, but also needs to spend less for capacity in the development stage. In this case, spending on development is less productive than research. This insight contradicts the myopic practice of giving priority to development resources in spending plans on grounds that profit is guaranteed from developed products. Our result shows that, in certain circumstances, research should be allocated more investment in spite of the uncertainty inherent to it.

The final case in which the inequalities in (27) and (29) are reversed gives rise to a balanced allocation of investments in that there is capacity for developing both types of projects.

Research output is often non-rival and partially excludable [20]. Higher quality research output has higher visibility and thus there is higher likelihood of essential knowledge of the project leaking to the community at large and also competitors. The leakage can have the effect of shrinking the gap between $\tilde{w}_l$ and $\tilde{w}_h$ and thereby contradicting condition (29). The laboratory may then spend less on research to implement the balanced strategy that develops all profitable projects. This influence of variability of project values on research funding is discussed further in the next section.

5 Numerical Case Studies

We report on results from various experiments conducted on the model. The results show in particular three possible pathways to a decline in research funding in industrial laboratories.

We consider an industrial laboratory with five development stages ($S = 5$). As a project moves along the stages, its state evolves according to a discretized Geometric Brownian Motion (GBM) process [21] specified as follows: at its inception at the research stage ($s = 0$), a project is at one of the two states in the set $\mathcal{K}_0 = \{0, 1\}$, with probabilities $1 - q$ and q respectively. There are three possible states associated with the first development stage, $\mathcal{K}_1 = \{0, 1, 2\}$. A project in state k ($k \in \mathcal{K}_0$) reaches either state k or $k + 1$ after the first-stage development, also with probabilities $1 - q$ and q respectively. At subsequent development stages, the sets of possible states are given by $\mathcal{K}_s = \{0, ..., s + 1\}$, and an incoming project in state k ($k \in \mathcal{K}_{s-1}$) reaches state k (with probability $1 - q$) or $k + 1$ (with probability q) after its development ($s = 2, \cdots, S$).

The expected revenues generated by an asset, w_{sk}, are given by (4). We assume that

$$r_{sk} = r_h^k r_l^{s+1-k},\ k \in \mathcal{K}_s = \{0, 1, ..., s+1\},\ s = 0, \cdots, S, \tag{34}$$

where r_h and r_l are two constants such that $r_h > 1 > r_l > 0$. Given the state evolution specified above, r_{sk} ($k \in \mathcal{K}_s$, $s = 0, \cdots, S$) is a discrete GBM process with $\bar{r} := qr_h + (1-q)r_l$ as both its mean starting value and the drift. We assume that the average lifetimes, μ_{sk} ($k \in \mathcal{K}_s$, $s = 0, \cdots, S$), are specific to stage but not state, i.e.,

$$\mu_{sk} = \mu_s \text{ for all } k \in \mathcal{K}_s \quad \text{where} \quad \sum_{s=0}^{S} \frac{1}{\mu_s} = 1.$$

after normalizing the total average lifetime of assets across different stages to unity. When $\mu_s = \infty$ ($s = 0, \cdots, S-1$) and $\mu_S = 1$, economic lifetimes of assets are 0 at all stages except the final stage, which is the only stage where revenue is generated. When μ_s ($s = 0, \cdots, S$) take other values, revenues are possibly generated in other stages.

Unless specified otherwise, we use as default values $q = 0.3$, $\gamma_s = 1$, and $c_{sk} = c = 0.2$ ($k \in \mathcal{K}_s, s = 1, \cdots, S$). Our findings are summarized below.

5.1 Lack of variability of project values leads to reductions in research spending

We let $\mu_s = \infty$ ($s = 0, \cdots, S-1$) and $\mu_S = 1$, so revenue is only generated in stage S. Since r_{sk} depends on r_{s-1k} ($k \in \mathcal{K}_s$, $s = 1, \cdots, S$), for $s < S$, r_{sk} can be viewed as an assessment at stage s of the revenue that the corresponding project may generate after it is fully developed. We keep the mean change of r_{sk} constant by letting $\bar{r} = 1.25$ and vary $\Delta r \doteq r_h - r_l$ to increase variability in projects' revenues generated at the final stage and their assessments at intermediate ones.

Table 1 shows changes of key outcomes with Δr. Both the optimal profit (Π) and the rate of return on investment (α) increase with Δr, consistent with findings in the literature that corporate R&D benefits from high variability in project values [8]. The fraction of projects generated at the research stage that complete the last-stage development, $\tilde{\theta}_S/\tilde{\theta}_0$, increases with decreasing Δr. Therefore we conclude that improvements in Π and α are achieved by being more selective in the gating decision; as illustration, note that in the case of the highest value of Δr, only a tiny fraction

(0.045) can survive to the last stage. Being selective means that only the most promising projects are developed, which implies that less development capacity is needed. Consequently, I_0/I, the percentage of the laboratory's budget invested in research, increases with Δr.

(r_l, r_h)	Δr	I_0/I	$\tilde{\theta}_S/\tilde{\theta}_0$	Π	α
(0.20, 3.70)	3.50	63.6%	0.045	36.34	1.82
(0.40, 3.23)	2.83	61.4%	0.059	26.34	1.32
(0.60, 2.77)	2.17	48.5%	0.153	19.77	0.99
(0.80, 2.30)	1.50	43.6%	0.197	14.45	0.72
(1.00, 1.83)	0.83	40.0%	0.300	11.03	0.55
(1.20, 1.37)	0.17	16.6%	1.000	9.38	0.47

Table 1: Project generation/completion rates and revenue, profit and return on investment, for different project value appreciation and depreciation rates

By the same token, we conclude that lower variability leads to less spending on research. In the table, as Δr drops from the highest level to the lowest level, the fraction of budget allocated to research drops from more than 60% to less than 20%.

5.2 Research spending may decrease as cost efficiency of development improves

We again assume that, by the choice of μ_s ($s = 0, \cdots, S$), all revenue is generated at the last stage, and focus on the cost-efficiency of project development. The efficiency improves if less investments are needed for the same amounts of processing capacities. This effect can be captured by lowering the investment per unit development capacity, γ_s ($s = 1, \cdots S$) in (15). In Figure 3, we set γ_s at the same value $\bar{\gamma}$ ($s = 1, \cdots, S$), vary the latter value, and show the resulting changes of I_0/I, the fraction of the budget spent on research.

Cost efficiency of project development also improves if a larger fraction of investment can be postponed to later development stages, so less investment is wasted on developing unsuccessful projects that are terminated before reaching the final stage. We capture this effect by letting

$$\gamma_s = \frac{\rho^s}{\sum_{i=1}^{S} \rho^i}\Gamma, \quad \rho \geq 0, s = 1, \cdots, S$$

where Γ is a constant that represents the required investment per unit of incremental capacity in all

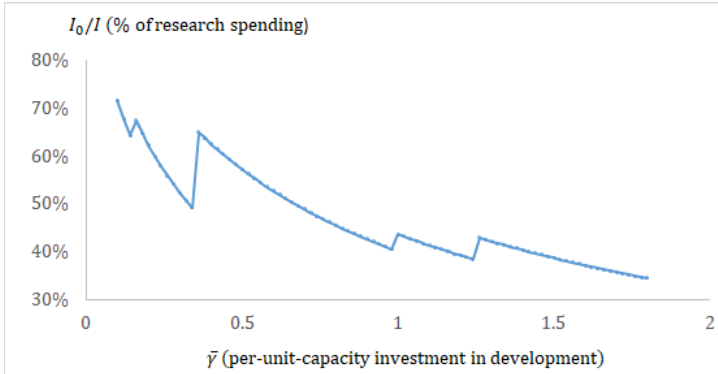

Figure 3: Impact of improving investment needed for development on research spending

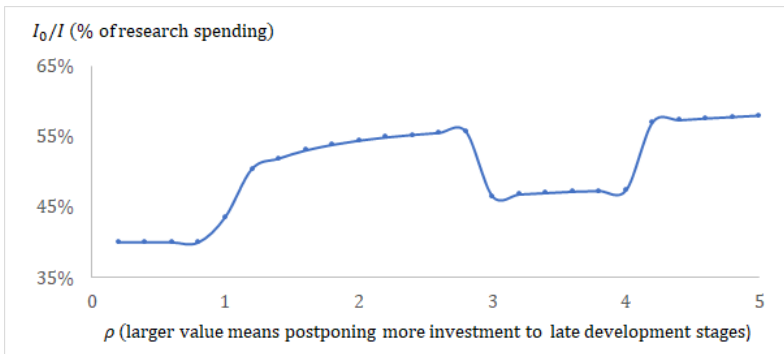

Figure 4: Impact of postponing investment to late development stages on research spending

development stages. Cost efficiency improves as ρ increases. When $\rho = 1$, the required investment per unit development capacity increment is uniform across all development stages. More investment per unit development capacity is needed at early stages if $\rho < 1$, and at late stages if $\rho > 1$. In Figure 4, we vary the value of ρ and show the resulting changes of I_0/I.

Both Figures 3 and 4 show a pattern of cyclic changes in the share of research spending, which characterizes the manager's optimal response to improvements in the cost efficiency of developing projects. In many cases, when the improvement takes place, the manager keeps the project gating decision unchanged while increasing research spending to generate more projects to "fill the pipe". However, when the accumulated improvement reaches a certain level, the manager may find it more profitable to start developing projects that would have been terminated previously. Funds are allocated to process these additional projects, causing the share of research spending to decline. Beyond this point, the research spending starts to increase under a new project gating decision, until when it becomes economical again to expand the range of eligible projects for development.

Cases in Figure 3 ($\bar{\gamma}$ decreases)			Cases in Figure 4 (ρ increases)		
$\bar{\gamma}$	$\tilde{\theta}_S/\tilde{\theta}_0$	I_0/I	ρ	$\tilde{\theta}_S/\tilde{\theta}_0$	I_0/I
0.6	0.3	52.7%	0.6	0.3	40.1%
0.5	0.3	57.5%	1	0.19	43.6%
0.4	0.3	62.5%	2	0.15	54.5%
0.3	0.51	52.3%	3	0.22	47.6%
0.2	0.51	62.2%	4	0.21	47.4%

Table 2: Key outcomes under changes of development investment. $\tilde{\theta}_S/\tilde{\theta}_0$: fraction of all projects developed end-to-end; I_0/I: research spending as a percentage of the total budget

Table 2 further illustrates this trend by showing values of I_0/I (the share of research spending) and $\tilde{\theta}_S/\tilde{\theta}_0$ (the percentage of projects that are developed end-to-end). The reduction of investment per unit of development capacity makes it profitable to let more projects reach the last stage, so $\tilde{\theta}_S/\tilde{\theta}_0$ is non-decreasing as $\bar{\gamma}$ is decreasing. When $\tilde{\theta}_S/\tilde{\theta}_0$ stays constant, I_0/I increases as $\bar{\gamma}$ decreases. When the reduction of $\bar{\gamma}$ causes $\tilde{\theta}_S/\tilde{\theta}_0$ to increase, the share drops significantly. In the other case, postponing investment to late development stages does not always lead to more projects to be fully developed, so $\tilde{\theta}_S/\tilde{\theta}_0$ may increase or decrease as ρ increases. However, I_0/I always decreases when $\tilde{\theta}_S/\tilde{\theta}_0$ increases significantly, which is consistent with our interpretation of Figure 4.

5.3 Research spending may decrease as more revenue is generated at early stages of development

In Table 3, we allow revenue to be generated in research and early development stages by choosing values of μ_s ($s = 0, \cdots, S$) (see Column 2 of the table). The table shows I_0/I, the fraction of budget spent on research, and optimal capacities of development stages.

As in the previous experiments, in case 1 revenue is generated only at the last stage. In contrast, in case 2 the asset lifetime is the same across all stages, so the same fraction of assessed revenue is realized at each stage. Not surprisingly, research spending is higher in the latter case.

Cases 3 and 4 are situations in which assets have non-zero lifetime at research, as well as early and late development stages. Thus revenue generation is concentrated at both ends of the R&D processes. In comparison with case 2, in which lifetimes are uniform across all stages, research spending may be higher (case 3) or lower (case 4).

The next four cases are the most interesting. In all cases, the research stage does not generate revenue. This can happen when research results, such as scientific discoveries and engineering principles, somehow become available to competitors, and hence have no economic lifetime of revenue generation. On the other hand, early-stage development can turn these results into revenue-generating assets, such as tools, algorithms, designs, and consulting software. Further development at late stages produces more assets, such as physical products, the lifetimes of which are subject to competition. From case 5 to 8, we increase lifetimes at early stages while decreasing those at late stages, letting assets generated by early development be the laboratory's main source of revenue. As such changes take place, the laboratory gives up late-stage developments. In case 5, development capacities are deployed at all five stages. In case 8, only the first two development stages are open.

Case	asset lifetime $(1/\mu_0, 1/\mu_1, 1/\mu_2, 1/\mu_3, 1/\mu_4, 1/\mu_5)$	I_0/I	Development Capacity				
			1	2	3	4	5
1	(0,0,0,0,0,1)	43.6%	2.62	2.62	2.62	1.72	1.72
2	(1/6, 1/6, 1/6, 1/6, 1/6, 1/6)	50.0%	3.00	3.00	1.53	1.53	0.91
3	(0.22, 0.14, 0.00, 0.14, 0.22, 0.28)	60.3%	3.02	1.08	1.08	1.08	1.08
4	(0.22, 0.11, 0.00, 0.11, 0.22, 0.33)	48.5%	2.91	2.91	1.49	1.49	1.49
5	(0.00, 0.20, 0.20, 0.20, 0.20, 0.20)	50.0%	3.00	3.00	1.53	1.53	0.91
6	(0.00, 0.57, 0.28, 0.11, 0.03, 0.00)	61.2%	3.69	3.69	0.33	0.10	0.00
7	(0.00, 0.69, 0.24, 0.06, 0.01, 0.00)	47.2%	9.45	0.85	0.26	0.00	0.00
8	(0.00, 0.76, 0.20, 0.04, 0.00, 0.00)	47.9%	9.57	2.87	0.00	0.00	0.00

Table 3: research spending (I_0/I) and development capacities adapted to changes of asset lifetimes and thus revenue generation

Interestingly, reducing the number of development stages does not always lead to more investment in research. To illustrate, in case 5 where all five development stages are open, half of the total budget is spent on research. In case 6, where 4 stages are open, the spending is more than 60%. However, the spending is below 50% in cases 7 and 8, which have three and two development stages respectively. The decline in the last two cases are accompanied by large increases in capacities deployed in early development stages, which suggests another reason for a decline in research spending. When more revenue can be obtained from asset banks at early stages, there will be little concern that a project will need to be terminated at subsequent development stages, and this will raise the incentive to invest more in early-stage development, at the expense of research.

6 Conclusions

We have developed a model of intellectual asset production in industrial laboratories. By analyzing its solutions, we have identified three possible pathways that can lead to laboratory manager to reduce the share of the budget that is invested in research. This can happen when projects lack sufficient differentiation in value, when investment in development capacities becomes more efficient; and when more revenues can be realized at early stages of development.

An obvious next step is to determine to what extent these possible reasons actually contribute to the observed decline in research's share in R&D funding. Making this assessment with our model requires a comparative analysis that will require investigating extensive empirical evidence based on past and present data of technology, market environment, and economic conditions. On the other hand, there already exists a rich set of empirical literature that has used survey data, patent records, and companies' equity values to estimate the distribution of value, economic lifetime, and profitability of intellectual assets produced by corporate R&D ([11], [12], [23]). We leave to the future the task of connecting these results with our model.

There is considerable scope for enhancements to our work. A major technical challenge is to go beyond the fluid scale and develop an efficient procedure to solve the model in its entirety as a stochastic control problem. This requires formulating project gating decisions as dynamic policies that adapt to random changes in project flow rates and available development capacities, both of which are stochastic processes subject to control exercised by budget allocations. Such extensions will deepen our understanding of incentives to invest in research in the presence of revenue uncertainty, and also contribute to improvements to the methodology for project management. Our model will also be enhanced by the inclusion of more general cost functions and more options in project management.

Our work also serves as a possible stepping stone towards a general model that addresses broader aspects of industrial R&D. Below we outline possible extensions.

First, the model assumes a fixed budget I, which in general is an investment made by the laboratory's parent company with expectations for a certain return on investment. The intrinsic return on investment α^* has a crucial role in this decision-making. A profit-seeking parent company typically sets the investment level based on the difference of α^* from the rate of return of investment from other assets. Extending our model to include such considerations will naturally lead to a dynamic endogenous growth model of industrial laboratories, which can then be used to assess long-term survivability and scale of operations in various business environments.

Second, the model also takes the expected lifetime value of assets as given parameters. In a broader framework, these values are influenced by the "leakage-effect" [20] as a consequence of which the laboratory may not be able to derive the full value from its output. From a societal point of view, the leakage speeds up propagation of new knowledge, even as it reduces the return on investment and hence the incentive to invest in industrial laboratories. Intellectual property management, such as patent regulations [12], allows policy-makers some degree of control of this trade-off. Our model here complements macroeconomic analysis of endogenous growth models ([19] (chapter 3), [20]) by providing an analytical framework for assessing the impact of national policy on industrial laboratories in the private sector.

We can envision even more ambitious enhancements to our model that break away from the currently assumed linear sequential structure of the multi-stage R&D process. In correspondence with the evolution of the technological and economic landscape, we can envision the need for new structures and models for organizing industrial laboratories, and their management. It will be useful, albeit challenging, to develop such models and also methods for their analysis, and, in particular, study the impact on the investment in research.

Appendix: Proofs

Proof of Proposition 1: The dual problem can be formulated thus:

$$\min_{\boldsymbol{\eta},\boldsymbol{\beta},\alpha\geq 0} \{\alpha I\} \tag{35}$$

subject to

$$\alpha\gamma_s \geq \eta_s, \quad 1 \leq s \leq S, \tag{36}$$

$$\alpha - \sum_{k\in\mathcal{K}_0} q_k^0 \beta_{1k} \geq \sum_{k\in\mathcal{K}_0} q_k^0 w_{0k}, \tag{37}$$

$$\beta_{sk} \geq \tilde{w}_{sk} - c_{sk} + \sum_{j\in\mathcal{K}_s} q_{kj}^s \beta_{s+1j} - \eta_s, \ k \in \mathcal{K}_{s-1}, \ 1 \leq s < S, \tag{38}$$

$$\beta_{Sk} \geq \tilde{w}_{Sk} - c_{Sk} - \eta_S, \quad k \in \mathcal{K}_{S-1}. \tag{39}$$

By (37), minimizing α requires minimizing $\boldsymbol{\beta}$ subject to (38) and (39) and that $\boldsymbol{\beta} \geq 0$. Thus

$$\begin{aligned}
\beta^*_{sk} &= (\tilde{w}_{sk} - c_{sk} + \sum_{j \in \mathcal{K}_s} q^s_{kj} \beta^*_{s+1j} - \eta^*_s)^+, \ k \in \mathcal{K}_{s-1}, \ s = 1, ..., S-1 \\
\beta^*_{Sk} &= (\tilde{w}_{Sk} - c_{Sk} - \eta^*_S)^+, \quad k \in \mathcal{K}_S.
\end{aligned} \tag{40}$$

If none of (36) and (37) is an equality, then we can reduce α^* to improve the objective value. If (36) are equalities for some s' ($1 \leq s' \leq S$) but (37) is not, then we can reduce $\eta^*_{s'}$ and α^* without violating (37). If (37) is an equality but (36) is not for some s' ($1 \leq s' \leq S$), then by (40), we can increase $\eta^*_{s'}$ to reduce β^*_{sk} ($k \in \mathcal{K}_{s-1}$, $1 \leq s' \leq S$) and α^*, until (36) becomes equality. Therefore, at the optimum, (36) and (37) are all equalities, which proves (19)-(20). Since β^*_{1k} ($k \in \mathcal{K}_0$) decreases in α^* by (21)-(22) and α^* strictly increases in β^*_{1k} ($k \in \mathcal{K}_0$) by (19), α^* is unique. ■

Proof of Proposition 2: The Proposition is immediate by observing that if $\alpha^* > 0$, then $\eta^*_s > 0$ ($s = 1, .., S$), and using the following complementary slackness conditions

$$\alpha^*(I - \theta_0 - \gamma_1 C^*_1 - .... - \gamma_S C^*_S) = 0$$

$$\eta^*_s(C^*_s - \sum_{k \in \mathcal{K}_{s-1}} \lambda^*_{sk}) = 0, \ s = 1, ..., S,$$

$$\beta^*_{1k}(\lambda^*_{1k} - q^0_k \theta_0) = 0, \ k \in \mathcal{K}_0$$

$$\beta^*_{sk}(\lambda^*_{sk} - \sum_{j \in \mathcal{K}_{s-2}} q^{s-1}_{jk} \lambda^*_{s-1j}) = 0, \ k \in \mathcal{K}_{s-1}, \ s = 2, ..., S,$$

$$\lambda^*_{sk}[\beta^*_{sk} - (\tilde{w}_{sk} - c_{sk} + \sum_{j \in \mathcal{K}_s} q^s_{kj} \beta^*_{s+1j} - \eta^*_s)] = 0, \ k \in \mathcal{K}_{s-1}, \ s = 1, ..., S-1,$$

$$\lambda^*_{Sk}[\beta^*_{Sk} - (\tilde{w}_{Sk} - c_{Sk} - \eta^*_S)] = 0, \ k \in \mathcal{K}_{S-1}. \quad ■$$

Proof of Corollary 1: Applying (19)-(22) in Proposition 1, the optimal solution in this case is

$$\alpha^* = \tilde{w}_0 + q\beta^*_h + (1-q)\beta^*_l, \ \beta^*_h = (\tilde{w}_h - c - \gamma\alpha^*)^+, \ \beta^*_l = (\tilde{w}_l - c - \gamma\alpha^*)^+.$$

If (27) is true, the above holds only if $\beta_h = \beta_l = 0$, which leads to (28). If (27) is false but (29) is true, then the above holds only if $\beta^*_h > 0$ and $\beta^*_l = 0$, which leads to (30)-(31). If neither (27) nor (29) is true, then above holds only if $\beta^*_h > 0$ and $\beta^*_l > 0$, which leads to (32)-(33). ■

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