

Modeling and Optimizing Intellectual Asset Production in Industrial Laboratories

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Abstract

We present a modeling framework to maximize value output of an industrial laboratory in which there is open-end research department that generates novel ideas and multiple development stages that commercialize some ideas into intellectual assets as well as final products. Resources control takes place at both tactical and strategic levels. The former decision is made at a fast time scale, and determines whether an individual idea should be advanced to a downstream development stage. As basis for this decision, the potential profitability of an idea is assessed by its option value instead of the expected return. The strategic management is carried out by long-term budget allocation between idea generation and development. We formulate and solve an optimization problem for determining the intrinsic rate of return of the laboratory. Our key finding is that the optimal allocation can be heavily biased towards idea generation, to the extent that its output overwhelms development resources. Such configuration inevitably causes a non-trivial number of generated ideas to be dropped, but the waste is appropriate for maximizing the total profit.

Keywords

Intellectual Asset, Innovation, Industrial Laboratories, rate of return

1. Introduction

Industrial laboratories are major institutional sources of innovations. In contrast to “open science” [1], which describes public-funded research conducted in universities and government laboratories, industrial laboratories conduct “commercial science”, where the mission is to generate profits for the parent company by creating, developing, and harvesting intellectual assets. These assets are incorporated in novel products and solutions, and patents. Society benefits, directly from the consumption and exercise of products and solutions, and the use through patent licensing of knowledge that is created, and indirectly from the spillover effect. Thus understanding the institution and the process is important at societal, national and company levels. This paper takes the first step by modeling and analysing the primary functions of industrial laboratories: the generation, development and harvesting of intellectual assets.

Our work is similar to model-based analysis in the supply chain literature in that both aim at optimizing business operations. Just as inventory modeling and optimization help supply chain managers to reduce their logistic costs and queueing modeling and optimization help service providers to improve their operations, our model provides insight for industrial laboratory managers to enhance efficiency of their R&D processes. Nevertheless, producing intellectual assets is a cumulative process that takes many stages to complete [2]. Unlike typical multi-stage productions, such as of conventional hardware and software, the production of intellectual assets involves high uncertainty. The output of the initial (research) stage is not the result of totally planned decisions, but rather a stream of possibilities of uncertain value and timing that require considerable custom development and leveraging, which also add uncertainty, before the value of the outputs can be ascertained and harvested. The output streams should therefore be considered to be stochastic processes. Outputs of downstream (development) stages are of various levels of quality, and reaching a

desired level is only partially within the control of the management. Also, unlike conventional products, intellectual asset production typically generates exploitable assets during the process. That is, an industrial laboratory does not need a project to reach the final marketable state for revenue to be generated through licensing of incremental advances, and indeed, the laboratory may accumulate a large portion of the total profit before the product goes to market.

The model and analysis in this paper are primarily concerned with operational issues in industrial laboratory management, including project selection and budget allocation. However, as we show here, the decisions made in this setting determine critical performance metrics, such as the intrinsic rate of return from investments made in industrial laboratory. These metrics have considerable input on strategic decision-making that determines long-term behavior. While acknowledging the limitations of the work here with its focus on the relatively short time-scale, we consider it as an essential building block of an extended model in a broader framework that addresses strategic and policy issues. These issues are touched upon in the final section and will be treated in future publications.

Below we present our model definition in Section 2, present a partial (fluid-scale) optimal solution in Section 3 and discuss managerial insights based on a two-stage process in Section 4 and Section 5 concludes.

2. Model

Our model for industrial laboratories is built from three basic elements: a multi-stage system representation of the end-to-end research and development processes; an information discovery process that progressively reveals potential values of projects; and a set of asset banks that contain value-generating but perishable assets such as patents.

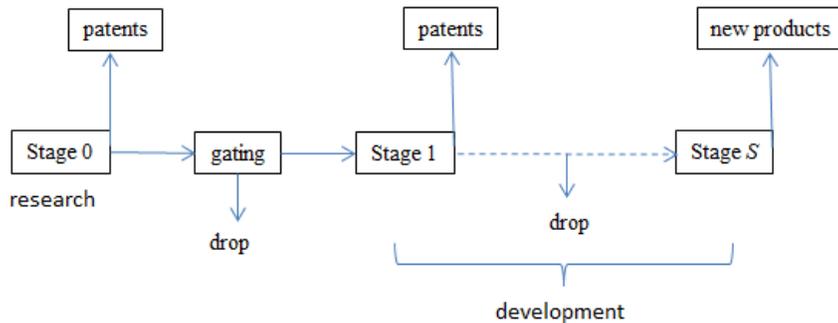


Figure 1: A Model of Industrial Laboratory

The multi-stage system is shown in Figure 1. The initial stage (stage 0) is where the open-ended research is conducted to create novel ideas. Each idea is the starting point of a potential project. A project can be transformed, implemented, and tested at subsequent development stages s ($s = 1, \dots, S$). The latter stages are staffed by development teams with specialized skills to deliver specific results. When a project reaches the final stage S , the idea has been developed into a marketable product. For instance, if the laboratory is in the software information technology (IT) industry, development stages can be mapped into planning, implementation, test, and release phases of software development. If the laboratory is in the pharmaceutical industry, some of the development stages correspond to different phases of clinical trials [2].

The value of an ongoing project is determined by the immediate value of its intellectual content and the potential of generating additional assets if the project continues to be processed at subsequent stages. The overall value is not known at the start but is increasingly ascertained as the project moves through the development stages. After each stage, a project goes through an assessment, which is used to determine whether it should be forwarded to the next stage for further development or terminated. To model value, we define a set of states that are specific to the stages. Projects belonging to the same state have the same assessed value. Initial stages have fewer states because there may not be enough information to distinguish values of projects that are in their infancy. The number of states grows as we proceed to latter stages, since more information about projects become available thus allowing a more comprehensive classification of values.

The R&D process is inherently uncertain. Not all projects that start from the same state at any stage will end up having the same value after processing at the next stage. On the other hand, it is reasonable to expect that the current state of

a project should have an important role in determining its state at the next stage. To capture both considerations, we denote the set of project states at stage s by \mathcal{K}_s ($s = 0, 1, \dots, S$), define q_k^0 as the probability that an idea generated from research is in state $k \in \mathcal{K}_0$ and q_{jk}^s as the probability that an incoming project in state j ($j \in \mathcal{K}_{s-1}$) will be in state k ($k \in \mathcal{K}_s$) after its development at stage s ($s = 1, \dots, S$). We denote vectors

$$\mathbf{q}^0 \equiv (q_1^0, \dots, q_{|\mathcal{K}_0|}^0) \quad \text{and} \quad \mathbf{q}_j^s \equiv (q_{j1}^s, \dots, q_{j|\mathcal{K}_s|}^s), \quad j \in \mathcal{K}_{s-1}.$$

The relationship of project values between different stages is shown in Figure 2. Observe that the above definition

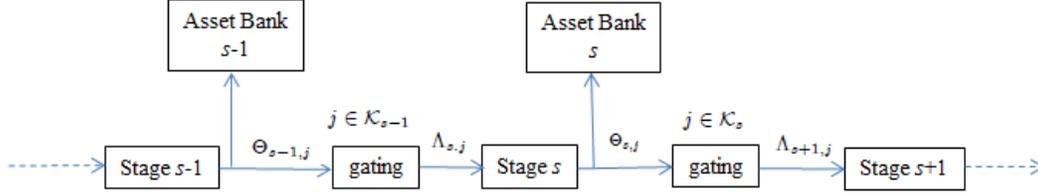


Figure 2: Variables at Stage s

implies that the evolution of values is Markovian: the output state of a project at a stage depends only on its state at the immediately preceding stage.

Value generation is reflected in our model by asset banks, defined at each stage for each state. One may consider an asset bank as a pool of intellectual assets that generate wealth over time, e.g., patents that can be licensed for fees or traded for other patents to avoid paying their holders. The asset bank at the last stage also includes new products that generate monopoly rent. The rate of value generated by an asset per unit of time, r_{sk} , depends on both the stage $k \in \mathcal{K}_s$ and state $s = 0, \dots, S$. Depending on circumstances, r_{sk} can be viewed as the average licensing fee per-unit of time per patent, or the average monopoly profit of a new product collected per-unit of time. The rate can also be zero if the corresponding bank holds worthless inventions or failed products. Like perishable inventories, intellectual assets lose their value over time. Hence each bank is associated with a birth-death process, where the “birth” corresponds to creation and “death” with cessation of the assets.

For each bank, we assume that the birth rate of its assets is proportional to the output rate of projects of the corresponding state, and without loss of generality, normalize the constant of proportionality to unity. The death rate is inversely proportional to the average economic lifetime of assets and denoted by μ_{sk} ($s \in \mathcal{K}_s$, $s = 0, 1, \dots, S$). Death is triggered by events like patent expiration or obsolescence, or in the case of the last stage, when the market of novel products is disrupted by new technologies. The birth-death process determines the equilibrium size of the asset bank, denoted by \mathcal{B}_{sk} ($k \in \mathcal{K}_s$, $s = 0, \dots, S$).

Managing industrial laboratories includes making two decisions: budget allocation and project gating. In the first decision, the manager determines the spending on each stage. Spending on the research stage is denoted by I_0 , which determines the rate of idea generation. While the actual volume of ideas generated per unit of time is stochastic, the rate is a deterministic increasing function of I_0 . Spending on each subsequent stage is denoted by I_s , which determines the processing capacity, C_s ($s = 1, \dots, S$). The total spending on all stages cannot exceed the corporate investment, I , the amount that the manager is authorized to spend, i.e.,

$$I_0 + \dots + I_S \leq I. \quad (1)$$

The exact form of $C_s(I_s)$ depends on the nature of development. For instance, if the development corresponds to clinical trials, then higher spending allows more trials to be conducted simultaneously but the duration of each trial cannot be shortened. Thus capacity increase is due to the increased number of projects that can be simultaneously developed, not the shorter development span of each project. On the other hand, in the case of software development where higher spending gives rise to a larger number of higher-quality teams, higher capacity is the result of both the increased number of simultaneous projects and a faster rate of development. For generality, here we only assume that $C_s(I_s)$ is increasing and concave (diminishing return of investment) in I_s .

The gating decision is made at each stage and formulated for steady state. Let Θ_0 be the number of ideas generated at stage 0 (research) per unit of time. Let Θ_{0k} be the number of these ideas assessed to be in state $k \in \mathcal{K}_0$. Then

$$\Theta_{0k} = \mathcal{M}_k(\Theta_0, \mathbf{q}^0), \quad k \in \mathcal{K}_0, \quad (2)$$

where $\mathcal{M}_k(\Theta, \mathbf{q})$ is the number of realizations of outcome k of a multinomial distribution with Θ as the number of trials and \mathbf{q} as the probability vector. For each unit of time, let Λ_{sj} be the number of projects in state j ($j \in \mathcal{K}_{s-1}$) that are admitted for development at stage s ($s = 1, \dots, S$) and Θ_{sk} be the number of projects completing their development at stage s ($s = 1, \dots, S$) and assessed to be in state $k \in \mathcal{K}_s$. Then the gating decision is captured by the condition that

$$0 \leq \Lambda_{sj} \leq \Theta_{s-1j}, \quad j \in \mathcal{K}_{s-1}, \quad s = 1, \dots, S, \quad (3)$$

i.e., of the projects that complete development at stage $s-1$, the manager selects some to proceed to stage s for further development. The selection is based on the project's assessed value (state). The total number of projects admitted into stage s ($s = 1, \dots, S$) is

$$\Lambda_s = \sum_{j \in \mathcal{K}_{s-1}} \Lambda_{sj}.$$

The numbers of processed projects in different states are

$$\Theta_{sk} = \sum_{j \in \mathcal{K}_{s-1}} \mathcal{M}_k(\Lambda_{sj}, \mathbf{q}_j^s) \quad k \in \mathcal{K}_s, \quad s = 1, \dots, S. \quad (4)$$

The gating capability is in part controlled by the allocated budget, which determines development capacity C_s at each stage s ($s = 1, \dots, S$). We formulate these constraints by

$$E[f(C_s, \Lambda_s)] \leq \bar{f}, \quad s = 1, \dots, S. \quad (4')$$

Given Λ_s ($s = 1, \dots, S$) as random variables, $f(\cdot)$ is a value function. For instance, if we model a development stage as a $M/M/1$ queueing system, and \bar{f} as the mean delay target that has to be met, then

$$E[f(C_s, \Lambda_s)] = (C_s - E[\Lambda_s])^{-1} \quad \text{where } E[\Lambda_s] < C_s.$$

In addition to capacity constraint, developing a project also incurs a direct cost, c_{sk} ($k \in \mathcal{K}_{s-1}$, $s = 1, \dots, S$).

Values are generated from asset banks at rates that are proportional to the bank sizes. We define v_{sk} as the rate for the bank of state k ($k \in \mathcal{K}_s$) and stage s ($s = 0, 1, \dots, S$). The net return from the investment in the laboratory is

$$\sum_{s=0}^S \sum_{k \in \mathcal{K}_s} v_{sk} \mathcal{B}_{sk} - \sum_{s=1}^S \sum_{k \in \mathcal{K}_{s-1}} c_{sk} \Lambda_{sk}. \quad (5)$$

To summarize, spending on research (I_0) determines the amount of ideas generated (Θ_0). Ideas of different qualities (Θ_{0k} ($k \in \mathcal{K}_0$)) determine the size of asset banks \mathcal{B}_{0k} . Applying project gating decision to Θ_{0k} determines the numbers of projects going into stage 1 (Λ_{1j} , ($k \in \mathcal{K}_0$)). The decision depends on I_1 in that Λ_{1j} is subject to capacity constraint at stage 1 ($C_1(I_1)$). Outputs of stage 1 (Θ_{1k} , ($k \in \mathcal{K}_1$)) determine the size of asset banks \mathcal{B}_{1k} ($k \in \mathcal{K}_1$), and after going through the next project gating decision, also the number of projects to be developed at the second stage per unit of time. Following this relationship in subsequent stages gives rise to \mathcal{B}_{2k} ($k \in \mathcal{K}_2$), ..., \mathcal{B}_{Sk} ($k \in \mathcal{K}_S$) and Λ_{2k} ($k \in \mathcal{K}_1$), ..., Λ_{Sk} ($k \in \mathcal{K}_{S-1}$), and to the profit in (5).

3. Fluid-Scale Optimization

In general, optimizing (5) is a stochastic control problem. In this paper, we optimize the problem at the fluid scale, which leads to a coarse solution that captures the first-order effects only. To this end, we define

$$\lambda_{sk} = E[\Lambda_{sk}] \quad (k \in \mathcal{K}_{s-1}) \quad \text{and} \quad \theta_{sk} = E[\Theta_{sk}] \quad (k \in \mathcal{K}_s)$$

which are the input and output rates of projects of states at stage s ($s = 1, \dots, S$) respectively, and

$$\lambda_s = E[\Lambda_s] \quad \text{and} \quad \theta_s = E[\Theta_s]$$

which are the total rates for stage s ($s = 1, \dots, S$). The output rates for the research stage, θ_{0k} ($k \in \mathcal{K}_0$) and θ_0 , are defined similarly. Since the gated output rate cannot exceed the input rate,

$$0 \leq \lambda_{sk} \leq \theta_{s-1k}, \quad k \in \mathcal{K}_{s-1}, \quad s = 1, \dots, S.$$

Following the Markovian formulation,

$$\theta_{sk} = \sum_{j \in \mathcal{K}_{s-1}} q_{jk}^s \lambda_{sj}, \quad k \in \mathcal{K}_s, \quad s = 1, \dots, S,$$

whereas at the initial (research) stage,

$$\theta_{0k} = q_k^0 \theta_0 \quad k \in \mathcal{K}_0.$$

We determine the expected size of asset banks,

$$B_{sk}(t) \equiv E[\mathcal{B}_{sk}(t)], \quad k \in \mathcal{K}_s, \quad s = 0, \dots, S$$

from the equilibrium condition of

$$\frac{dB_{sk}(t)}{dt} = \theta_{sk} - \mu_{sk} B_{sk}(t) = 0, \quad k \in \mathcal{K}_s, \quad s = 0, \dots, S, \quad (6)$$

which gives

$$B_{sk} = \frac{\theta_{sk}}{\mu_{sk}}, \quad k \in \mathcal{K}_S, s = 0, \dots, S, \quad (7)$$

for all t in equilibrium. Hence the rate of value generation at each bank in equilibrium is

$$v_{sk} B_{sk} = v_{sk} \frac{\theta_{sk}}{\mu_{sk}} \quad k \in \mathcal{K}_S, s = 0, \dots, S. \quad (8)$$

Since v_{sk} denotes the value of intellectual asset generated per unit of time and μ_{sk} is the inverse of the expected lifetime of assets,

$$w_{sk} \equiv v_{sk} / \mu_{sk}, \quad k \in \mathcal{K}_S, s = 0, \dots, S,$$

is the expected value of an intellectual asset accumulated over its entire economic life. Hence (8) can be written alternatively as

$$v_{sk} B_{sk} = w_{sk} \theta_{sk}, \quad k \in \mathcal{K}_S, s = 0, \dots, S,$$

i.e., the product of the expected lifetime value of an asset and the rate at which these assets flow into the bank.

To complete our definition of the fluid model, we assume the particularly simple case of a linear relationship between the processing capacity and investment at each stage s , i.e.,

$$C_s = \gamma_s I(s), \quad 1 \leq s \leq S.$$

We also assume that the idea generation rate θ_0 is linearly related to the investment, $\theta_0 = \gamma_0 I_0$. It is straightforward to accommodate a generalized capacity constraint in the development stage, such as following 4', $(C_s - E[\Lambda_s])^{-1} \leq \bar{f}$, where \bar{f} is a given positive parameter. In this simple case, the optimization problem below remains intact with only minor redefinition of C_s and γ_s , $s = 1, \dots, S$. With these specifications, the fluid-scale optimization problem is formulated as

$$\max_{\underline{\lambda}, \underline{\theta}, C \geq 0} \left\{ \sum_{s=0}^S \sum_{k \in \mathcal{K}_S} w_{sk} \theta_{sk} - \sum_{s=1}^S \sum_{k \in \mathcal{K}_{S-1}} c_{sk} \lambda_{sk} \right\} \quad (9)$$

$$\text{s. t.} \quad \gamma_0 \theta_0 + \gamma_1 C_1 + \dots + \gamma_S C_S \leq I \quad (10)$$

$$\lambda_{s+1k} \leq \theta_{sk}, \quad k \in \mathcal{K}_S, s = 0, \dots, S-1. \quad (11)$$

$$\theta_{sk} = \sum_{j \in \mathcal{K}_{S-1}} q_{jk}^s \lambda_{sj}, \quad k \in \mathcal{K}_S, s = 1, \dots, S. \quad (12)$$

$$\sum_{k \in \mathcal{K}_{S-1}} \lambda_{sk} \leq C_s, \quad s = 1, \dots, S. \quad (13)$$

$$\theta_{0k} = q_k^0 \theta_0, \quad k \in \mathcal{K}_0. \quad (14)$$

Applying (12) to the first term in (9), the expected value generated from all assets banks at stage s ($s = 1, \dots, S$)

$$\sum_{k \in \mathcal{K}_S} w_{sk} \theta_{sk} = \sum_{k \in \mathcal{K}_S} w_{sk} \sum_{j \in \mathcal{K}_{S-1}} q_{jk}^s \lambda_{sj} = \sum_{j \in \mathcal{K}_{S-1}} \tilde{w}_{sj} \lambda_{sj}$$

where

$$\tilde{w}_{sj} \equiv \sum_{k \in \mathcal{K}_S} w_{sk} q_{jk}^s, \quad j \in \mathcal{K}_{S-1}, s = 1, \dots, S$$

can be viewed as the expected (immediate) asset value obtained from processing a state- j project at a single stage, namely s . Without loss of generality, we normalize γ_0 to 1 and transform the above fluid model as follows

$$\max_{\underline{\lambda}, \theta_0, C \geq 0} \left\{ \sum_{k \in \mathcal{K}_0} w_{0k} q_k^0 \theta_0 + \sum_{s=1}^S \sum_{k \in \mathcal{K}_{S-1}} (\tilde{w}_{sk} - c_{sk}) \lambda_{sk} \right\} \quad (15)$$

$$\text{s. t.} \quad \theta_0 + \gamma_1 C_1 + \dots + \gamma_S C_S \leq I \quad (16)$$

$$\lambda_{1k} \leq q_k^0 \theta_0 \quad (k \in \mathcal{K}_0) \quad (17)$$

$$\lambda_{sk} \leq \sum_{j \in \mathcal{K}_{S-2}} q_{jk}^{s-1} \lambda_{s-1j} \quad k \in \mathcal{K}_{S-1}, s = 2, \dots, S \quad (18)$$

$$\sum_{k \in \mathcal{K}_{S-1}} \lambda_{sk} \leq C_s \quad s = 1, \dots, S. \quad (19)$$

The insights of the fluid model are more obvious from examining its dual formulation. Let α , β_{1k} ($k \in \mathcal{K}_0$), β_{sk} ($k \in \mathcal{K}_{S-1}, s = 2, \dots, S$), and η_s ($s = 1, \dots, S$) be the dual variables associated with constraints (16), (17), (18), and (19), respectively. We interpret α as the intrinsic rate of return of the laboratory; η_s as the shadow price of processing capacity of stage s ($s = 1, \dots, S$), and β_{sk} as the option value of a project in state k ($k \in \mathcal{K}_S$) after its development at stage

$s - 1$ ($s = 1, \dots, S$), i.e., the gain that can be expected by continuing the development of the project at stage s instead of terminating it¹. The relationships of these quantities are given by the following lemma.

Proposition 1. *The optimal dual solution of (15)-(19) is given as follows*

$$\alpha^* = \sum_{k \in \mathcal{X}_0} q_k^0 (w_{0k} + \beta_{1k}) \quad (20)$$

$$\eta_s^* = \alpha^* \gamma_s, \quad s = 1, \dots, S \quad (21)$$

$$\beta_{Sk}^* = (\tilde{w}_{Sk} - c_{Sk} - \alpha^* \gamma_S)^+, \quad k \in \mathcal{X}_{S-1} \quad (22)$$

$$\beta_{sk}^* = (\tilde{w}_{sk} - c_{sk} + \sum_{j \in \mathcal{X}_S} q_{kj}^s \beta_{s+1j} - \alpha^* \gamma_s)^+, \quad k \in \mathcal{X}_{S-1}, \quad 1 \leq s < S, \quad (23)$$

where α^* is a unique value.

Proof. The dual problem can be formulated as

$$\min_{\eta, \beta, \tau, \alpha \geq 0} \{\alpha I\} \quad (24)$$

$$\text{s. t.} \quad \alpha \gamma_s \geq \eta_s, \quad 1 \leq s \leq S, \quad (25)$$

$$\alpha - \sum_{k \in \mathcal{X}_0} q_k^0 \beta_{1k} \geq \sum_{k \in \mathcal{X}_0} q_k^0 w_{0k}, \quad (26)$$

$$\beta_{sk} \geq \tilde{w}_{sk} - c_{sk} + \sum_{j \in \mathcal{X}_S} q_{kj}^s \beta_{s+1j} - \eta_s, \quad k \in \mathcal{X}_{S-1}, \quad 1 \leq s < S, \quad (27)$$

$$\beta_{Sk} \geq \tilde{w}_{Sk} - c_{Sk}, \quad k \in \mathcal{X}_{S-1} - \eta_S, \quad k \in \mathcal{X}_S. \quad (28)$$

To minimize α , (27) and (28) should hold at equality unless the right-hand side is strictly negative, in which case it is optimal to let $\beta_{sk} = 0$ ($k \in \mathcal{X}_{S-1}, s = 1, \dots, S$). Thus

$$\beta_{sk}^* = (\tilde{w}_{sk} - c_{sk} + \sum_{j \in \mathcal{X}_S} q_{kj}^s \beta_{s+1j}^* - \eta_s^*)^+, \quad k \in \mathcal{X}_{S-1}, \quad s = 1, \dots, S - 1$$

$$\beta_{Sk}^* = (\tilde{w}_{Sk} - c_{Sk} - \eta_S^*, \quad k \in \mathcal{X}_S)^+ \quad (29)$$

To prove (20)-(22), we need to show that both (25) and (26) are binding, which is true because otherwise α^* will not be the minimum value: if neither (25) nor (26) is binding, then we obviously can reduce α^* . If (26) is non-binding but (25) is binding for some s' ($s' = 1, \dots, S$), then a reduction of $\eta_{s'}^*$ that keeps (26) intact strictly reduces α . If (26) is binding and (25) is non-binding for some s' ($s' = 1, \dots, S$), then (29) shows that a slight increase in $\eta_{s'}^*$ reduces β_{sk}^* ($k \in \mathcal{X}_{S-1}, s = 1, \dots, s'$) and thus α^* .

Finally to prove α^* is unique, if $\beta_{1k} = 0$ for all $k \in \mathcal{X}_0$, then $\alpha^* = 0$. Otherwise (22)-(23) indicate that β_{1k}^* ($k \in \mathcal{X}_0$), if strictly positive, strictly decreases in α^* . The uniqueness follows from applying the fixed-point theorem to (20). \square

An interpretation of the proposition is that the system is driven by the intrinsic rate of return α^* : by (21), the shadow cost of development capacity (η_s^*) is this rate multiplied by the capacity per unit of investment. By (22), projects at the final stage are assessed by the difference of their immediate asset value (\tilde{w}_{Sk}) from the direct development cost (c_{Sk}) and the shadow cost of development capacity. The excess of this difference over the intrinsic rate of return is the option value, which is zero if the difference does not exceed the intrinsic rate. By (23), the assessment of projects at the earlier stages should also include option values of the project at the subsequent stage. Moreover, (20) indicates that the rate is intrinsic because it is determined endogenously as the expected return (immediate assets + option value) of ideas generated at the initial stage.

The optimal dual solution naturally provides the optimal policy for gating in project management, as implied by the primal solutions of the fluid-scale optimization. The policy, given in the following proposition, indicates that projects with zero option values should be terminated. The investment on the development capacity should be at the level that can handle all admitted projects without leaving any idle capacity.

¹Our option value is not exactly the same as that commonly defined in the real option literature because we are not assuming the presence of a complete financial market.

Proposition 2. Assume the non-trivial case where $\alpha^* > 0$. Then following solution optimizes the primal fluid model,

$$\theta_0^* = I - \gamma_1 C_1^* - \dots - \gamma_S C_S^* \quad (30)$$

$$C_s^* = \sum_{k \in \mathcal{K}_{s-1}} \lambda_{ks}^*, s = 1, \dots, S \quad (31)$$

$$\lambda_{sk}^* = \left(\sum_{j \in \mathcal{K}_{s-2}} q_{jk}^{s-1} \lambda_{s-1j}^* \mathbf{1}(\beta_{sk}^* > 0) \right) \quad k \in \mathcal{K}_{s-1}, s = 1, \dots, S \quad (32)$$

The Proposition is immediate from applying the assumption that $\alpha^* > 0$ (so $\eta_s^* > 0, s = 1, \dots, S$) and the following complementary slackness conditions

$$\begin{aligned} \alpha^* (I - \theta_0 - \gamma_1 C_1^* - \dots - \gamma_S C_S^*) &= 0, \\ \eta_s^* (C_s^* - \sum_{k \in \mathcal{K}_{s-1}} \lambda_{ks}^*) &= 0, \quad s = 1, \dots, S, \\ \beta_{sk}^* (\lambda_{sk}^* - \sum_{j \in \mathcal{K}_{s-2}} q_{jk}^{s-1} \lambda_{s-1j}^*) &= 0, \quad k \in \mathcal{K}_{s-1}, s = 1, \dots, S, \\ \lambda_{sk}^* [\beta_{sk}^* - (\tilde{w}_{sk} - c_{sk} + \sum_{j \in \mathcal{K}_s} q_{kj}^s \beta_{s+1j}^* - \eta_s^*)] &= 0, \quad k \in \mathcal{K}_{s-1}, s = 1, \dots, S-1 \\ \lambda_{Sk}^* [\beta_{Sk}^* - (\tilde{w}_{Sk} - c_{Sk} - \eta_S^*)] &= 0, \quad k \in \mathcal{K}_{S-1}. \end{aligned}$$

The following are some important insights: there can be cases where $\beta_{sj}^* = 0$ for all $j \in \mathcal{K}_s$ at some stage $s < S$. Such a solution suggests that no project should be developed beyond stage $s-1$ and the laboratory has its income generated solely from asset banks of early stages $(0, 1, \dots, s-1)$. By doing so, the laboratory becomes a high-end ‘knowledge factory’ that engages only in early incubation. In other cases, if there is an outside supplier (university or start-up, for instance), which is willing to sell the parent company comparable innovations at the price less than α^* , then the laboratory’s research is not competitive and the laboratory should close its research arm and be a development shop that concentrates on product realization. There can also be cases where the parent company’s cost of capital is higher than α^* . Hence from the financial perspective, the laboratory is not contributing enough to its parent company to justify its continuing existence.

4. A System with Two Stages: Research and Development

We consider the simplest case and derive managerial insights from the corresponding model. Consider a laboratory that has two stages, research and a single development stage. Ideas generated from research belongs to one of two states, high or low value states, i.e., $\mathcal{K}_0 = \{h, l\}$. The probability of a newly-generated idea being in each state is

$$q_h^0 = q \text{ and } q_l^0 = 1 - q.$$

The expected (immediate) asset value of these ideas is

$$V_0 = q w_{0h} + (1 - q) w_{0l}$$

For simplicity, we assume that at the development stage, the direct cost is the same for both types of projects, i.e., $c_{1h} = c_{1l} = c$. The expected values of developing these projects are

$$\tilde{w}_h \equiv \sum_{k \in \mathcal{K}_1} q_{hk}^1 w_{0k}, \text{ and } \tilde{w}_l \equiv \sum_{k \in \mathcal{K}_1} q_{lk}^1 w_{0k}.$$

We apply the solution from the fluid-scale optimization in Section 3 to this simple model. For this simple setting we omit some indices and denote β_h and β_l as option values for developing projects in states h and l , and λ_h and λ_l as the rates of admitting these projects for development respectively. We denote C as the development capacity and γ as the amount of required investment per unit of additional capacity. The optimal solution is the following corollary of Propositions 1 and 2.

Corollary 1. It is optimal to invest the entire budget in research if and only if

$$V_0 > \frac{\tilde{w}_h - c}{\gamma}, \quad (33)$$

in which case

$$\begin{aligned} \alpha^* &= V_0, \quad \beta_h^* = \beta_l^* = 0 \\ \text{and } \theta_0^* &= I, \quad \lambda_h^* = \lambda_l^* = C^* = 0. \end{aligned} \quad (34)$$

Otherwise, the optimal solution depends on the condition

$$\frac{V_0 + q(\tilde{w}_h - c)}{1/\gamma + q} > \tilde{w}_l - c. \quad (35)$$

If the above inequality holds, then the optimal solution of the dual problem is

$$\beta_l^* = 0, \beta_h^* = \frac{\tilde{w}_h - c - \gamma V_0}{1 + q\gamma}, \alpha^* = \frac{V_0 + q(\tilde{w}_h - c)}{1 + q\gamma}, \quad (36)$$

and that of the primal problem is

$$\lambda_l^* = 0, \lambda_h^* = C^* = \frac{q}{1 + q\gamma}I, \theta_0^* = \frac{I}{1 + q\gamma}. \quad (37)$$

If the inequality in (35) does not hold, the optimal dual solution is

$$\begin{aligned} \beta_l^* &= \frac{(\tilde{w}_l - c) - q\gamma(\tilde{w}_h - \tilde{w}_l) - \gamma V_0}{1 + \gamma}, \\ \beta_h^* &= \frac{(\tilde{w}_h - c) + (1 - q)\gamma(\tilde{w}_h - \tilde{w}_l) - \gamma V_0}{1 + \gamma}, \\ \alpha^* &= \frac{V_0 - c + q\tilde{w}_h + (1 - q)\tilde{w}_l}{1 + \gamma}, \end{aligned} \quad (38)$$

and that of the primal problem is

$$\lambda_l^* = \frac{1 - q}{1 + \gamma}I, \lambda_h^* = \frac{q}{1 + \gamma}I, \theta_0^* = C^* = \frac{I}{1 + \gamma}. \quad (39)$$

Proof. By Proposition 1, the dual solution is in one of the three cases: If $\beta_h^* = \beta_l^* = 0$, then $\alpha^* = V_0$.

If $\beta_h^* > 0$ and $\beta_l^* = 0$, then

$$\begin{aligned} \alpha^* &= V_0 + q\beta_h^*, \\ \beta_h^* &= \tilde{w}_h - c - \gamma\alpha^*, \end{aligned}$$

leads to the solution in (36)-(37).

If $\beta_l^* > 0$, then

$$\begin{aligned} \alpha^* &= V_0 + q\beta_h^* + (1 - q)\beta_l^* \\ \beta_h^* &= \tilde{w}_h - c - \gamma\alpha^*, \\ \beta_l^* &= \tilde{w}_l - c - \gamma\alpha^*, \end{aligned}$$

which leads to the solution in (38)-(39).

Since

$$\begin{aligned} \alpha^* &= V_0 + q\beta_h^* + (1 - q)\beta_l^*, \\ \beta_h^* &= \tilde{w}_h - c - \alpha^*\gamma, \end{aligned}$$

$\beta_h^* = 0$ and (34) applies if and only if

$$\tilde{w}_h - c - \gamma\alpha^* < 0$$

when $\alpha^* = V_0$, which is (34).

Now suppose that (33) does not hold. If (35) holds, then

$$\frac{(\tilde{w}_l - c) - q\gamma(\tilde{w}_h - \tilde{w}_l) - \gamma V_0}{1 + \gamma} = \frac{(1 + q\gamma)(\tilde{w}_l - c) - \gamma(q(\tilde{w}_h - c) + V_0)}{1 + \gamma} < 0,$$

in which case (38) does not apply since it would lead to $\beta_l^* < 0$. Consequently (36) is the only possibility, which also gives rise to (37).

If (35) does not hold, then

$$\tilde{w}_l - c - \gamma \frac{q(\tilde{w}_h - c) + \gamma V_0}{1 + q\gamma} \geq 0.$$

Using α^* in (36),

$$\beta_l^* = \tilde{w}_l - c - \gamma\alpha^* \geq 0,$$

so (36) does not apply, making (38), and thus (39), the only possibility. \square

Condition (33) specifies a special regime in which the intellectual contents generated from research is so valuable that it becomes more profitable to eliminate development and let the laboratory become a pure idea factory.

There are situations where the laboratory may not derive much immediate gains from under-developed and untested ideas (i.e., V_0 is small so (33) cannot hold). It then becomes appropriate for the laboratory to remain an end-to-end

system. For instance, the solution in (36)-(37) conveys an important managerial insight: for parameters in the region specified by (35), resource allocation is unbalanced, i.e., more ideas are generated from the research stage than there is capacity in the downstream stages to develop the ideas. Hence inevitably ideas in the lower-value state will be dropped, even when they can generate a positive return i.e., $\tilde{w}_l - c > 0$.

This phenomenon reflects the different nature of research and development: the value of research output is unknown in advance, so for each idea not generated, the laboratory loses an average value of a potential project. Once generated, assessments can be made to classify projects into different states. By first dropping projects in the lower-value states, the manager loses the below-average value. Hence spending on development is inherently less productive compared to research. This insight contradicts myopic tendency to prioritize investment on development projects with ensured profit. Here we show that uncertainty should be the reason to attract more investment.

Assuming small V_0 , so that the laboratory will not give up development, condition (35) requires q to be large, and \tilde{w}_h to significantly exceed \tilde{w}_l , which implies that the unbalanced resource allocation should be attempted only if the management can always select higher-potential projects. Whether this can be achieved depends not only on the manager's intrinsic ability but also on the external environment. For instance, research output is often non-rival and partially excludable. The high quality idea has higher visibility and thus higher probability to be leaked to the outside. The leakage can have the effect of shrinking the gap between values \tilde{w}_l and \tilde{w}_h . In this case, the laboratory can be better off by implementing a more balanced strategy that ensures the processing of all profitable new ideas (i.e., adopting solution (38)-(39)).

5. Conclusions

As alluded to in the Introduction, in addition to giving operational insights, the development in this paper also serves as a stepping stone towards a general model that addresses broader aspects of industrial laboratories. To conclude this paper, we outline the following possible extensions.

First, the model assumes a fixed budget I , which in general is an investment made by the laboratory's parent company with expectations for a certain rate of return. The intrinsic rate of return α^* serves a crucial role in this decision-making. A profit-seeking parent company typically sets the investment level based on the difference of α^* from the rate of return of investment from other assets. Extending our model to include such considerations will naturally lead to an endogenous growth model of industrial laboratories, which can then be used to assess long-term survivability and scale of operations in various business environments.

Second, the model also takes the expected lifetime value of assets, v , as given parameters. Under a broader consideration, these values are influenced by the "leakage-effect" [6] by which the laboratory may not be able derive the full value from this output. From a societal point of view, the leakage speeds up propagation of new knowledge, while on the other hand, it also reduces the rate of return and hence the incentive to invest in industrial laboratories. Intellectual property management, such as patent regulations [3], [4], allows policy-makers some degree of control of this trade-off. Our model here complements macroeconomic analysis of endogenous growth models [5] (chapter 3), [6] by providing an analytical framework for assessing the impact of policy on laboratory operations.

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