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e-companion
ONLY AVAILABLE IN ELECTRONIC FORM

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Proofs, Construction of Numerical Examples in §5 and Supplement to §7

Some of our results assume a Cobb-Douglas quality function, as defined in (6), which satisfies

$$q(\tau,x)=x^{\alpha}\tau^{\beta},\ q_{\tau}=\frac{\beta q(\tau,x)}{\tau},\ \text{and } q_x=\frac{\alpha q(\tau,x)}{x}. \tag{EC.1}$$

We will begin with two lemmas that are relevant to such results, then prove all propositions stated in the paper, prove that the duopoly equilibrium characterized by (15) remains a sequential equilibrium when $\gamma=0$, $\alpha+\beta=1$ and we give each firm the option of introducing arbitrarily many new products in a row, and finally give details on the construction of the numerical examples in §5.

Lemma 1 *The monopoly equilibrium development time τ^m is a unique solution to*

$$c=qf\left[1-\beta\frac{f(\tau)}{\tau}-\alpha\right]=\left[\alpha f\left(\tau\right)\right]^{\frac{\alpha}{1-\alpha}}f\left(\tau\right)\tau^{\frac{\beta}{1-\alpha}}\left[1-\beta\frac{f\left(\tau\right)}{\tau}-\alpha\right] \tag{EC.2}$$

and the equilibrium expenditure and quality improvement per new product are given by

$$x^m=\left[\alpha f\left(\tau^m\right)\right]^{\frac{1}{1-\alpha}}\left(\tau^m\right)^{\frac{\beta}{1-\alpha}},\ q(\tau^m,x^m)=\left[\alpha f(\tau^m)\right]^{\frac{\alpha}{1-\alpha}}\left(\tau^m\right)^{\frac{\beta}{1-\alpha}}$$

$$\textit{Thus } x^m=\alpha qf \textit{ and } \frac{dx^m}{d\tau^m}=\frac{\alpha}{1-\alpha}\left(q_{\tau}f+qf_{\tau}\right). \tag{EC.3}$$

Proof:

By Proposition 1 (for general quality function, proved below), the monopoly equilibrium (τ^m,x^m) is the unique solution:

$$q_{\tau}(\tau,x)f(\tau)-q(\tau,x)+\frac{x+c}{f}=0 \tag{EC.4}$$

$$q_x(\tau,x)f(\tau)=1. \tag{EC.5}$$

Substituting the expression for q_x in (EC.1) into (EC.5),

$$q_x(\tau^m,x^m)f(\tau^m)=\frac{\alpha q(\tau^m,x^m)}{x^m}f(\tau^m)=\alpha\frac{(\tau^m)^{\beta}}{(x^m)^{\alpha-1}}f(\tau^m)=1, \tag{EC.6}$$

and (EC.3) follows immediately by solving the last equality in (EC.6) for x^m and $q(\tau^m,x^m)=(x^m)^{\alpha}(\tau^m)^{\beta}$, respectively. Substituting the expression for q_{τ} in (EC.1) and the expressions for x and q in (EC.3) into (EC.4) establishes (EC.2). ■

Lemma 2 *The duopoly equilibrium development time τ^d is the unique solution to*

$$\begin{aligned} c &= \frac{q}{\delta}\left[1-\beta\frac{f(\tau)}{(1+\gamma)\tau}-\alpha\delta f(\tau)\right] \\ &= \frac{(\alpha f(\tau))^{\frac{\alpha}{1-\alpha}}}{\delta}[(1+\gamma)\tau]^{\frac{\beta}{1-\alpha}}\left[1-\beta\frac{f(\tau)}{(1+\gamma)\tau}-\alpha\delta f(\tau)\right] \end{aligned} \tag{EC.7}$$

and the duopoly equilibrium expenditure and quality improvement per new product are given by

$$x^d = [\alpha f(\tau^d)]^{\frac{1}{1-\alpha}}[(1+\gamma)\tau^d]^{\frac{\beta}{1-\alpha}},\ q((1+\gamma)\tau^d,x^d)=[\alpha f(\tau^d)]^{\frac{\alpha}{1-\alpha}}[(1+\gamma)\tau^d]^{\frac{\beta}{1-\alpha}}$$

$$\textit{Thus } x^d = \alpha q f \textit{ and } \frac{dx^d}{d\tau^d}=\frac{\alpha}{1-\alpha}\left(\frac{\partial q}{\partial \tau}f+qf_\tau\right)=\frac{\alpha}{1-\alpha}((1+\gamma)q_\tau f+qf_\tau) \tag{EC.8}$$

where

$$\frac{\partial q}{\partial \tau}=(1+\gamma)q_\tau((1+\gamma)\tau,x)=\beta\frac{q}{\tau}.$$

Proof

By Proposition 4 (for general quality function, proved below), the duopoly equilibrium is symmetric and the unique solution to:

$$q_\tau((1+\gamma)\tau,x)f(\tau)-q((1+\gamma)\tau,x)+\delta(x+c)=0 \tag{EC.9}$$

$$q_x((1+\gamma)\tau,x)f(\tau)=1 \tag{EC.10}$$

Substituting the expression for q_x in (EC.1) into (EC.10) and using $q((1+\gamma)\tau^d,x^d)=(x^d)^\alpha((1+\gamma)\tau^d)^\beta$,

$$q_x((1+\gamma)\tau^d,x^d)f(\tau^d)=\alpha\frac{(x^d)^\alpha[(1+\gamma)\tau^d]^\beta}{x^d}f(\tau^d)=1, \tag{EC.11}$$

and (EC.8) follows immediately by solving the last equality in (EC.11) for x^d and inserting the solution into the expression of q. Using $x^d=\alpha qf$ and $q_\tau=\beta f/((1+\gamma)\tau^d)$ in (EC.9), and then substituting the expression for q from (EC.1),

$$c=\frac{[-q_\tau((1+\gamma)\tau^d,x^d)f(\tau^d)+q((1+\gamma)\tau^d,x^d)-\delta x^d]}{\delta}$$

$$= \frac{q((1+\gamma)\tau^d, x^d)}{\delta}\left[1 - \frac{\beta f(\tau^d)}{(1+\gamma)\tau^d} - \alpha\delta f(\tau^d)\right] \tag{EC.12}$$
$$= \frac{[\alpha f(\tau^d)]^{\frac{\alpha}{1-\alpha}}[(1+\gamma)\tau^d]^{\frac{\beta}{1-\alpha}}}{\delta}\left[1 - \beta\frac{f(\tau^d)}{(1+\gamma)\tau^d} - \alpha\delta f(\tau^d)\right]$$

which establishes τ^d is the unique solution to (EC.7). ■

Proof of Proposition 1: The equilibrium (τ^m, x^m) is characterized by three conditions: discounted profit maximization by the monopolist, stationarity, and rational expectations, expressed as follows:

$$(\tau^m, x^m) = \arg\max_{\tau\geq 0, x\geq 0}\{e^{-\delta\tau}[q(\tau,x)f(\widetilde{\tau}) - x - c + \pi^m]\}, \tag{EC.13}$$

$$\pi^m = e^{-\delta\tau_m}[q(\tau^m, x^m)f(\widetilde{\tau}) - x^m - c + \pi^m], \tag{EC.14}$$

$$\widetilde{\tau} = \tau^m. \tag{EC.15}$$

The monopolist can obtain zero discounted profit by setting $\tau^m = \infty$. Therefore, we can rule out a boundary solution with $\tau^m = 0$ or $x^m = 0$, which would result in negative discounted profit. Consequently, any solution to (EC.13)-(EC.15) must satisfy the first-order conditions

$$e^{-\delta\tau}\left[q_\tau(\tau,x)f - q(\tau,x) + \frac{x+c}{f}\right] = 0 \tag{EC.16}$$

$$q_x(\tau,x)f(\tau) = 1. \tag{EC.17}$$

For any (τ, x) that satisfies (EC.16) and (EC.17), τ must be a solution to

$$H_m(\tau) \equiv q_\tau(\tau, x(\tau))f(\tau) - q(\tau, x(\tau)) + \frac{x(\tau)+c}{f(\tau)} = 0 \tag{EC.18}$$

where $x(\tau)$ is the unique solution to (EC.17). It follows that

$$\lim_{\tau\to\infty} H_m(\tau) = -\lim_{\tau\to\infty}\frac{1}{f(\tau)}\left[q(\tau, x(\tau))f(\tau) - x(\tau) - c\right].$$

By assumption, there exists some τ_0 such that $q(\tau_0, x(\tau_0))f(\tau_0) - x(\tau_0) - c > 0$. Because $q(\tau, x(\tau))f(\tau) - x(\tau) - c$ increases in τ, the above limit is negative. Moreover

$$\lim_{\tau\to 0} H_m(\tau) \geq \lim_{\tau\to 0}\frac{c}{f(\tau)} > 0,$$

so there exists some τ^m such that $H_m(\tau^m) = 0$. To show τ^m is unique, we prove $H_m(\tau)$ strictly decreases with τ as follows

$$
\begin{aligned}
\frac{dH_m(\tau)}{d\tau} &= \frac{\partial H_m}{\partial \tau} + \frac{\partial H_m}{\partial x}\frac{dx}{d\tau} \\
&= q_{\tau\tau} f + q_\tau f_\tau - q_\tau - \frac{x+c}{f^2} f_\tau + \left(q_{\tau x} f - q_x + \frac{1}{f}\right)\frac{dx}{d\tau} \\
&= -q_\tau(1-f_\tau) + q_{\tau\tau} f - \frac{(x+c)q_x}{f} f_\tau + q_{\tau x} f \frac{dx}{d\tau} \quad \text{because } q_x f = 1 \\
&< q_{\tau\tau} f - \frac{x q_x}{\tau} f_\tau + q_{\tau x} f \frac{dx}{d\tau} \quad \text{because } f_\tau = e^{-\delta\tau} \in (0,1) \text{ and } \tau > f(\tau) \\
&= q_{\tau\tau} f - \frac{x q_x}{\tau} f_\tau + q_{\tau x} f \left(\frac{q_{\tau x} f + q_x f_\tau}{-q_{xx} f}\right) \\
&= \frac{q_{\tau\tau} q_{xx} - q_{\tau x}^2}{q_{xx}} f - \left(\frac{x}{\tau} - \frac{q_{\tau x}}{|q_{xx}|}\right) q_x f_\tau \\
&\le 0 \quad \text{by (4) and (5).}
\end{aligned}
\tag{EC.19}
$$

Furthermore, given τ^m is unique, our assumption $q_{xx} < 0$ implies that x^m is uniquely determined by the first-order condition $q_x(\tau^m, x) f(\tau^m) = 1$.

From (EC.13) and (EC.16), the monopoly profit at the equilibrium is

$$
\pi^m(\tau^m, x^m) = \frac{e^{-\delta\tau^m}}{(1-e^{-\delta\tau^m})}[q(\tau^m, x^m) f(\tau^m) - x^m - c] = \frac{e^{-\delta\tau^m} q_\tau(\tau^m, x^m) f^2(\tau^m)}{(1-e^{-\delta\tau^m})} > 0. \quad \blacksquare
$$

Proof of Proposition 2: Let $\pi^m(\tau, x)$ denote the monopolist's discounted profit, the objective function in (12). The first-order conditions $\partial\pi^m/\partial\tau = 0$ and $\partial\pi^m/\partial x = 0$ imply, respectively, that an optimal solution (τ^*, x^*) to (12) must satisfy:

$$
q_\tau f + q f_\tau - q + \frac{x^* + c}{f} = 0 \text{ and } q_x f = 1. \tag{EC.20}
$$

Substituting (EC.20) into the definition (EC.18) of $H_m(\tau)$, we have

$$
H_m(\tau^*) = q_\tau f - q + \frac{x^* + c}{f} = q_\tau f - q - (q_\tau f + q f_\tau - q) = -q f_\tau < 0. \tag{EC.21}
$$

Then, because $H_m(\tau^m) = 0$ and $H_m(\tau)$ strictly decreases in τ, we conclude that $\tau^m < \tau^*$. Then $x^m < x^*$ follows from the first-order conditions $q_x(\tau^*, x^*) f(\tau^*) = 1$ and $q_x(\tau^m, x^m) f(\tau^m) = 1$, $f_\tau > 0$, $q_{xx} < 0$, and $q_{\tau x} \ge 0$.

Let $S^m(\tau, x)$ denote the sum of profit and consumer surplus, the objective function in (13). The first-order conditions

$$\frac{\partial S^m}{\partial \tau} = -\frac{\delta e^{-\delta\tau}}{(1-e^{-\delta\tau})^2}(\frac{q}{\delta} - x - c) + \frac{e^{-\delta\tau}}{(1-e^{-\delta\tau})}\frac{q_\tau}{\delta} = 0$$

and $\partial S^m/\partial x = 0$ imply, respectively, that an optimal solution (τ^{**}, x^{**}) to (13) must satisfy

$$q_\tau f - q + \delta(x+c) = 0 \text{ and } q_x = \delta. \tag{EC.22}$$

For any (τ, x) that satisfies (EC.22), τ must be a solution to

$$G_m(\tau) \equiv q_\tau(\tau, x^s(\tau))f(\tau) - q(\tau, x^s(\tau)) + \delta(x^s(\tau) + c) = 0,$$

where $x^s(\tau)$ is the unique solution to $q_x(\tau, x) = \delta$. We will show that the function $G_m(\tau)$ strictly decreases with τ, which implies that τ^{**} is the unique solution to $G_m(\tau) = 0$. By substituting $q_x = \delta$ and then, using the implicit function theorem, $dx^s/d\tau = -q_{\tau x}/q_{xx}$, we find that

$$\begin{aligned}\frac{dG_m(\tau)}{d\tau} &= q_{\tau\tau}f + q_\tau f_\tau - q_\tau + (q_{\tau x}f - q_x + \delta)\frac{dx^s}{d\tau} \\ &< \left(q_{\tau\tau} + q_{\tau x}\frac{dx^s}{d\tau}\right)f \\ &= \left(\frac{q_{\tau\tau}q_{xx} - q_{\tau x}^2}{q_{xx}}\right)f \\ &< 0.\end{aligned}$$

As $G_m(\tau^{**}) = 0$, to prove that $\tau^{**} < \tau^m$ it remains to show that $G_m(\tau^m) < 0$. Let x^s denote $x^s(\tau^m)$. Observe that $x^s > x^m$ because $q_x(\tau^m, x^m) = 1/f(\tau^m) > \delta = q_x(\tau^m, x^s)$ and $q_{xx} < 0$. That is, holding the equilibrium development time τ^m fixed, the equilibrium expenditure x^m is strictly lower than the level that would maximize the sum of profit and consumer surplus. It follows that

$$\begin{aligned}G(\tau^m) &= G(\tau^m) - H(\tau^m) \\ &< [q_\tau(\tau^m, x^s) - q_\tau(\tau^m, x^m)]f(\tau^m) - [q(\tau^m, x^s) - q(\tau^m, x^m)] + \delta x^s - \frac{x^m}{f(\tau^m)} \\ &= f(\tau^m)\int_{x^m}^{x^s} q_{\tau x}(\tau^m, x)dx - \int_{x^m}^{x^s} q_x(\tau^m, x)dx + x^s q_x(\tau^m, x^s) - x^m q_x(\tau^m, x^m) \\ &= f(\tau^m)\int_{x^m}^{x^s} q_{\tau x}(\tau^m, x)dx + \int_{x^m}^{x^s} x q_{xx}(\tau^m, x)dx\end{aligned}$$

$$< \int_{x^m}^{x^s} [\tau^m q_{\tau x}(\tau^m, x) + x q_{xx}(\tau^m, x)] dx$$

$$\leq 0 \qquad \text{by assumption (5).}$$

This completes the proof that $\tau^{**} < \tau^m$. ■

Proof of Proposition 3: Using $\delta f = 1 - f_\tau$ in (EC.9), at $((1+\gamma)\tau^d, x^d)$,

$$q_\tau f - q f_\tau = \delta(qf - x - c). \tag{EC.23}$$

For a duopolist to be profitable, the right hand side of (EC.23) must be positive. With a Cobb-Douglas quality function, $q_\tau = \beta q/((1+\gamma)\tau^d)$, so

$$q_\tau f - q f_\tau = q\left[\frac{\beta}{(1+\gamma)\tau^d} f - f_\tau\right] = q f_\tau \left[\frac{\beta(e^{\delta\tau^d} - 1)}{(1+\gamma)\delta\tau^d} - 1\right] > 0. \tag{EC.24}$$

The duopolists are profitable if and only if

$$\tau^d > z_0/\delta \quad \text{where } z_0 \text{ is the unique positive solution to } \beta(e^z - 1) = (1+\gamma)z. \tag{EC.25}$$

Because τ^d strictly increases in c (see the proof Proposition 6 for the general case), and $\delta\tau^d > z_0$ implies a lower bound on c_d^l where c_d^l is obtained (EC.7) with $\tau^d = z/\delta$, i. e.,

$$c_d^l = \frac{[\alpha(1-e^{-z})]^{\alpha/(1-\alpha)}}{\delta^{(1+\beta)/(1-\alpha)}}[(1+\gamma)z]^{\frac{\beta}{1-\alpha}}\left[1 - \alpha(1-e^{-z}) - \beta\frac{1-e^{-z}}{(1+\gamma)z}\right], \tag{EC.26}$$

and c_d^l is strictly positive because

$$\left[1 - \alpha(1-e^{-z}) - \frac{\beta(1-e^{-z})}{(1+\gamma)z}\right] = 1 - \alpha(1-e^{-z}) - \frac{(1-\alpha)(1-e^{-z})}{(1+\gamma)z} > 0. \tag{EC.27}$$

For given δ, $\tau^d \leq z_0/\delta$ if and only if $c \leq c_d^l$, so c_d^l is the threshold for non-negative duopoly profit. Note that z_0 is invariant with respect to δ, and therefore (EC.26) implies that c_d^l decreases in δ.■

Proof of Proposition 4: For simplicity, we omit the superscript d and let $\{\tau_i, x_i\}_{i=1,2}$ denote the duopoly equilibrium. The first-order optimality conditions for (15) and our assumption that the firms earn positive profit at the duopoly equilibrium imply that:

$$q(\gamma\tau_{-i} + \tau_i, x_i) - \delta x_i - q_\tau(\gamma\tau_{-i} + \tau_i, x_i) f(\tau_{-i}) = \delta c, \tag{EC.28}$$

$$q_x(\gamma\tau_{-i}+\tau_i,x_i)f(\tau_{-i})=1, \tag{EC.29}$$

$$q(\gamma\tau_{-i}+\tau_i,x_i)f(\tau_{-i})-x_i-c>0. \tag{EC.30}$$

We take two steps to prove that the duopoly equilibrium is symmetric: $\tau_1=\tau_2$ and $x_1=x_2$.

Step 1: We prove that if $\tau_1>\tau_2$ then $x_1\geq x_2$. Define

$$J(x|\tau)\equiv q_\tau(\tau,x)-[q(\tau,x)-\delta(x+c)]q_x(\tau,x). \tag{EC.31}$$

By (EC.28) and (EC.29), $J(x_1|\gamma\tau_2+\tau_1)=0$ and $J(x_2|\gamma\tau_1+\tau_2)=0$. Because $\gamma\in[0,1]$, $q_\tau>0$, $q_{\tau\tau}<0$ and $q_{\tau x}\geq 0$, the condition $\tau_1>\tau_2$ implies that

$$J(x_2|\gamma\tau_2+\tau_1)\leq J(x_2|\gamma\tau_1+\tau_2)=0. \tag{EC.32}$$

We next prove that

$$\frac{dJ(x|\gamma\tau_2+\tau_1)}{dx}>0 \text{ for all } x\in[x_1,\infty) \text{ such that } J(x|\gamma\tau_2+\tau_1)=0, \tag{EC.33}$$

so $x_1\geq x_2$ must be true. Otherwise we would have a contradiction, as $x_1<x_2$ and (EC.32) would imply existence of some $x\in[x_1,x_2]$ such that $J(x|\gamma\tau_2+\tau_1)=0$ and $dJ(x|\gamma\tau_2+\tau_1)/dx\leq 0$. To show (EC.33), note that by (EC.29) and (EC.30)

$$(x_1+c)<\frac{q(\gamma\tau_2+\tau_1,x_1)}{q_x(\gamma\tau_2+\tau_1,x_1)}. \tag{EC.34}$$

Because $q_{xx}<0$, $q(.,x)-(x+c)q_x(.,x)$ increases in x, so (EC.34) applies to all $x\in[x_1,\infty)$.

Substituting the left hand side of (EC.34) for $(x+c)$ in (EC.31), for $x\in[x_1,\infty)$, if $J(x|\gamma\tau_2+\tau_1)=0$, then

$$q_x(\gamma\tau_2+\tau_1,x)-\delta<\frac{q_\tau(\gamma\tau_2+\tau_1,x)}{q(\gamma\tau_2+\tau_1,x)}. \tag{EC.35}$$

$$\begin{aligned}\frac{dJ(x|\gamma\tau_2+\tau_1)}{dx}&=q_{\tau x}-q_{xx}[q-\delta(x+c)]-q_x(q_x-\delta)\\&>q_{\tau x}-q_{xx}[q-\delta(x+c)]-\frac{q_\tau}{q}q_x \quad \text{by (EC.35)}\\&=q_{\tau x}+\frac{|q_{xx}|q_\tau}{q_x}-\frac{q_\tau}{q}q_x \quad \text{because } J(x|\gamma\tau_2+\tau_1)=0\end{aligned}$$

$$= q_\tau(\frac{q_{\tau x}}{q_\tau} + \frac{|q_{xx}|}{q_x} - \frac{q_x}{q})$$
$$> 0 \text{ by (1) and (16).}$$

Step 2: We now show that if $\tau_1 > \tau_2$ and $x_1 \geq x_2$, then (EC.28) cannot hold for both $i = 1,2$. Since

$$\begin{aligned}
&[q(\gamma\tau_2+\tau_1,x_1)-\delta x_1]-[q(\gamma\tau_1+\tau_2,x_2)-\delta x_2] \\
&\geq [q(\gamma\tau_2+\tau_1,x_1)-q(\gamma\tau_2+\tau_1,x_2)]-\delta(x_1-x_2) \\
&\geq [q(\gamma\tau_2+\tau_1,x_1)-q(\gamma\tau_2+\tau_1,x_2)]-\delta\frac{q(\gamma\tau_2+\tau_1,x_1)-q(\gamma\tau_2+\tau_1,x_2)}{q_x(\gamma\tau_2+\tau_1,x_1)} \text{ because } q_{xx}<0 \\
&= [q(\gamma\tau_2+\tau_1,x_1)-q(\gamma\tau_2+\tau_1,x_2)][1-\delta f(\tau_2)] \\
&\geq 0.
\end{aligned} \tag{EC.36}$$

Let $\chi(\tau)$ be the implicit function defined by $q_x(\tau,x)f(\tau_2)=1$, holding τ_2 constant. It follows that

$$\frac{d\chi(\tau)}{d\tau} = -\frac{q_{\tau x}(\tau,\chi(\tau))}{q_{xx}(\tau,\chi(\tau))} \text{ and } x_1 = \chi(\gamma\tau_2+\tau_1).$$

Together, $q_x(\gamma\tau_1+\tau_2,\chi(\gamma\tau_1+\tau_2))f(\tau_2) = q_x(\gamma\tau_1+\tau_2,x_2)f(\tau_1)=1$, $f(\tau_2)<f(\tau_1)$ and $q_{xx}<0$ imply that

$$x_2 > \chi(\gamma\tau_1+\tau_2).$$

It follows, using $q_{\tau x} \geq 0$, that

$$q_\tau(\gamma\tau_1+\tau_2,\chi(\gamma\tau_1+\tau_2)) \leq q_\tau(\gamma\tau_1+\tau_2,x_2),$$

and therefore

$$\begin{aligned}
&q_\tau(\gamma\tau_2+\tau_1,x_1)f(\tau_2)-q_\tau(\gamma\tau_1+\tau_2,x_2)f(\tau_1) \\
&\leq [q_\tau(\gamma\tau_2+\tau_1,x_1)f(\tau_2)-q_\tau(\gamma\tau_1+\tau_2,\chi(\gamma\tau_1+\tau_2))f(\tau_1) \\
&< f(\tau_1)[q_\tau(\gamma\tau_2+\tau_1,\chi(\gamma\tau_2+\tau_1))-q_\tau(\gamma\tau_1+\tau_2,\chi(\gamma\tau_1+\tau_2))] \\
&= f(\tau_1)\int_{\gamma\tau_1+\tau_2}^{\gamma\tau_2+\tau_1}\left[q_{\tau\tau}(\tau,\chi(\tau))+q_{\tau x}(\tau,\chi(\tau))\frac{d\chi(\tau)}{d\tau}\right]d\tau \\
&= f(\tau_1)\int_{\gamma\tau_1+\tau_2}^{\gamma\tau_2+\tau_1}\frac{q_{\tau\tau}(\tau,\chi(\tau))q_{xx}(\tau,\chi(\tau))-q_{\tau x}^2(\tau,\chi(\tau))}{q_{xx}(\tau,\chi(\tau))}d\tau
\end{aligned}$$

$$\leq 0 \text{ because } q_{\tau\tau}q_{xx} \geq q_{\tau x}^2 \text{ and } q_{xx} < 0. \tag{EC.37}$$

Combining (EC.36) and (EC.37),

$$q(\gamma\tau_2+\tau_1,x_1)-\delta x_1-q_\tau(\gamma\tau_2+\tau_1,x_1)f(\tau_2)>q(\gamma\tau_1+\tau_2,x_2)-\delta x_2-q_\tau(\gamma\tau_1+\tau_2,x_2)f(\tau_1),$$

so (EC.28) cannot hold for both $i=1,2$. This contradiction completes Step 2 and establishes that the equilibrium must be symmetric.

We now show that the symmetric equilibrium $\tau^d \equiv \tau_1 = \tau_2$, and $x^d \equiv x_1 = x_2$ is the unique equilibrium. Motivated by the first-order conditions ($EC.28$) and ($EC.29$), define

$$H_d(\tau) \equiv q_\tau((1+\gamma)\tau, x(\tau))f(\tau) - q((1+\gamma)\tau, x(\tau)) + \delta x(\tau) + \delta c, \tag{EC.38}$$

where $x(\tau)$ is the solution to $q_x((1+\gamma)\tau, x(\tau))f(\tau)=1$,so $H_d(\tau^d)=0$ and

$$\begin{aligned}
\frac{dH_d(\tau)}{d\tau} &= -(1+\gamma-f_\tau)q_\tau + (1+\gamma)q_{\tau\tau}f + (q_{\tau x}f - q_x + \delta)\frac{dx}{d\tau} \\
&< (1+\gamma)q_{\tau\tau}f + (q_{\tau x}f - q_x + \delta)\frac{(1+\gamma)q_{\tau x}f + q_x f_\tau}{-q_{xx}f} \\
&= (1+\gamma)q_{\tau\tau}f + (q_{\tau x}f - q_x + q_x\delta f)\frac{(1+\gamma)q_{\tau x}f + q_x f_\tau}{-q_{xx}f} \\
&= (1+\gamma)\left(\frac{q_{\tau\tau}q_{xx} - q_{\tau x}^2}{q_{xx}}\right)f + \left[\gamma\frac{q_{\tau x}q_x}{q_{xx}} + \frac{q_x^2 f_\tau}{q_{xx}f}\right]f_\tau \text{ because } f_\tau = 1-\delta f \\
&< 0,
\end{aligned}$$

i.e., $H_d(\tau)$ is strictly decreasing in τ. Therefore, τ^d is unique. Furthermore, given the unique τ^d, our assumption $q_{xx}<0$ implies that x^d is uniquely determined by the first-order condition $q_x(\tau^d, x)f(\tau^d) = 1$. ■

Proof of Proposition 5: Let $\pi^d(\tau, x)$ denote the duopolists' profit, which is the objective function in (17). At the optimum (τ^*, x^*) in (17),

$$\frac{\partial \pi^d}{\partial \tau} = \frac{e^{-\delta\tau}}{(1-e^{-\delta\tau})^2}[\delta(x^*+c) - \delta q f + (1-f_\tau)((1+\gamma)q_\tau f + q f_\tau)] = 0. \tag{EC.39}$$

Recall the definition of $H_d(\tau)$ from (EC.38). Evaluating $H_d(\tau)$ at τ^* and observing that $x(\tau^*) = x^*$ (for brevity, we leave out arguments $((1+\gamma)\tau^*, x^*)$ in q_τ and q, and τ^* in f and f_τ),

$$H_d(\tau^*) = q_\tau f - q + \delta(x^* + c)$$

$$= q_\tau f - q + \delta q f - (1 - f_\tau)[(1+\gamma)q_\tau f + q f_\tau]$$

$$= [(1+\gamma)f_\tau - \gamma]q_\tau f - (2 - f_\tau)q f_\tau$$

$$< 0,$$

where the second and third equality employ (EC.39) and $\delta f = 1 - f_\tau$, respectively. To understand the final inequality, note that by (EC.39),

$$[\delta f - (1 - f_\tau)f_\tau]q = (1 - f_\tau)(1+\gamma)q_\tau f \ + \delta(x_d^* + c).$$

The left hand side is $(1 - f_\tau)^2 q$ and $\delta(x_d^* + c) > 0$, so the above equality implies that

$$(1 - f_\tau)q > (1+\gamma)q_\tau f.$$

Therefore

$$(2 - f_\tau)q f_\tau > (1 - f_\tau)q f_\tau > (1+\gamma)q_\tau f f_\tau \geq [(1+\gamma)f_\tau - \gamma]q_\tau f.$$

Since $H_d(\tau)$ is strictly decreasing and $H_d(\tau^d) = 0$, it must be that $\tau^d < \tau^*$. Then $x^d < x^*$ follows from the first-order conditions $q_x((1+\gamma)\tau^d, x^d)f(\tau^d) = 1$ and $q_x((1+\gamma)\tau^*, x^*)f(\tau^*) = 1$, $f_\tau > 0$, $q_{xx} < 0$, and $q_{\tau x} \geq 0$.

The sum of profit and consumer surplus (18):

$$S^d(\tau, x) = \frac{e^{-\delta\tau}}{1 - e^{-\delta\tau}}\left[\frac{q((1+\gamma)\tau, x)}{\delta} - x - c\right].$$

An optimal solution (τ_d^{**}, x_d^{**}) to (18) must satisfy the first-order conditions:

$$\frac{\partial S^d}{\partial \tau} = \frac{e^{-\delta\tau}}{(1 - e^{-\delta\tau})^2}[-q((1+\gamma)\tau, x) + \delta(x + c) + (1+\gamma)q_\tau((1+\gamma)\tau, x)f(\tau)] = 0$$
$$\frac{\partial S^d}{\partial x} = \frac{e^{-\delta\tau}}{(1 - e^{-\delta\tau})}\left[\frac{q_x((1+\gamma)\tau, x)}{\delta} - 1\right] = 0. \tag{EC.40}$$

Given τ, let $x^s(\tau)$ denote the unique solution to (EC.40), so that

$$\frac{dx^s}{d\tau} = -(1+\gamma)\frac{q_{\tau x}}{q_{xx}}.$$

From the first equation of (EC.40),

$$G_d(\tau) \equiv -q((1+\gamma)\tau, x^s(\tau)) + \delta(x^s(\tau) + c) + (1+\gamma)q_\tau((1+\gamma)\tau, x^s(\tau))f(\tau)$$

satisfies $G_d(\tau^{**})=0$. Because $q_{\tau\tau}q_{xx}\ge q_{\tau x}^2$ and $q_{xx}<0$,

$$\begin{aligned}\frac{dG_d(\tau)}{d\tau}&=-(1+\gamma)q_\tau+(1+\gamma)q_\tau f_\tau+(1+\gamma)^2q_{\tau\tau}f+[-q_x+\delta+(1+\gamma)q_{\tau x}f]\frac{dx^s}{d\tau}\\&=-(1+\gamma)(1-f_\tau)q_\tau+(1+\gamma)^2q_{\tau\tau}f+(1+\gamma)q_{\tau x}\frac{dx^s}{d\tau}\\&<(1+\gamma)^2\left(q_{\tau\tau}-\frac{q_{\tau x}^2}{q_{xx}}\right)f\qquad\text{because } dx^s/d\tau=-(1+\gamma)q_{\tau x}/q_{xx}\\&\le 0.\end{aligned}$$

Therefore, in order to prove that $\tau^d<\tau^{**}$, it will be sufficient to show that $G_d(\tau^d)>0$.

For brevity, let q^s denote $q((1+\gamma)\tau^d,x^s(\tau^d))$, let q_x^s denote $q_x((1+\gamma)\tau^d,x^s(\tau^d))$, let q_τ^s denote $q_\tau((1+\gamma)\tau^d,x^s(\tau^d))$, and let x^s denote $x^s(\tau^d)$. Then using the second equation of (EC.40)

$$q_x^s=\delta$$

and with a Cobb-Douglas formulation (EC.1)

$$\begin{aligned}G_d(\tau^d)&=-q^s+\delta(x^s+c)+(1+\gamma)q_\tau^s f\\&=-q^s+\delta\frac{\alpha q^s}{q_x^s}+\delta c+(1+\gamma)\frac{\beta}{(1+\gamma)\tau^d}q^s f\\&=q^s\left(\alpha-1+\beta\frac{f}{\tau^d}\right)+\delta c\end{aligned}$$

For brevity, let q^d denote $q((1+\gamma)\tau^d,x^d)$, where x^d is duopoly equilibrium. Because

$$q_x^d f=\frac{[(1+\gamma)\tau^d]^\beta}{(x^d)^{1-\alpha}}\frac{1-e^{-\delta\tau^d}}{\delta}=1 \text{ and } q_d^s=\frac{[(1+\gamma)\tau^d]^\beta}{(x^s)^{1-\alpha}}=\delta,$$

given the same τ^d

$$\frac{x^d}{x^s}=\left(1-e^{-\delta\tau^d}\right)^{\frac{1}{1-\alpha}} \text{ and } \frac{q^d}{q^s}=\left(\frac{x^d}{x^s}\right)^\alpha=(1-e^{-\delta\tau^d})^{\frac{\alpha}{1-\alpha}}.$$

Let $z=\delta\tau^d$ and define

$$\eta(\alpha,z)=\frac{1}{q^s}G_d(\frac{z}{\delta})-(1-e^{-z})^{\frac{1}{1-\alpha}}\frac{\beta\gamma}{(1+\gamma)z}.$$

Using (EC.7) for δc

$$\eta(\alpha,z)=\alpha-1+\beta\frac{f}{\tau^d}+\frac{q^d}{q^s}\left[1-\frac{\beta f}{(1+\gamma)\tau^d}-\alpha\delta f\right]-(1-e^{-z})^{\frac{1}{1-\alpha}}\frac{\beta\gamma}{(1+\gamma)z}$$

$$= \alpha - 1 + \beta \frac{f}{\tau^d} + (1 - e^{-\delta\tau^d})^{\frac{1}{1-\alpha}} \left[\frac{1}{1 - e^{-\delta\tau^d}} - \frac{\beta}{(1+\gamma)\delta\tau^d} - \alpha - \frac{(1-\alpha)\gamma}{(1+\gamma)z} \right]$$
$$= \alpha - 1 + \beta \frac{1 - e^{-z}}{z} + (1 - e^{-z})^{\frac{1}{1-\alpha}} \left[\frac{1}{1 - e^{-z}} - \alpha - \frac{\beta}{z} \right].$$

Observe that if $\forall \alpha \in [0,1)$, $\eta(\alpha, z) > 0$ for all $z > 0$, then $G_d(\tau^d) > 0$. To show $\eta(\alpha, z) > 0$, first note $\eta(0, z) = 0$ for all z and

$$\frac{\partial \eta}{\partial z}|_{\alpha=0} = -\beta \frac{1 - e^{-z}}{z^2} + \beta \frac{e^{-z}}{z} + (1 - e^{-z}) \left[-\frac{e^{-z}}{(1 - e^{-z})^2} + \frac{\beta}{z^2} \right] + e^{-z} \left[\frac{1}{1 - e^{-z}} - \frac{\beta}{z} \right]$$
$$= 0$$
$$\frac{\partial \eta}{\partial \alpha}|_{\alpha=0} = 1 - (1 - e^{-z}) - \frac{1}{(1-\alpha)^2} (1 - e^{-z}) \ln(1 - e^{-z}) \left(\frac{1}{1 - e^{-z}} - \frac{\beta}{z} \right)$$
$$= e^{-z} - \left(1 - \beta \frac{1 - e^{-z}}{z} \right) \ln(1 - e^{-z}) > 0,$$
$$\text{so } \frac{d\eta}{d\alpha}|_{\alpha=0} = \frac{\partial \eta}{\partial \alpha}|_{\alpha=0} + \frac{\partial \eta}{\partial z} \frac{\partial z}{\partial \alpha}|_{\alpha=0} > 0.$$

It follows that for $\alpha \in (0, \varepsilon)$ and ε sufficiently small, $\eta(\alpha, z) > 0$, so $G_d(\tau^d) > H_d(\tau^d)$, and therefore we conclude that $\tau^d < \tau^{**}$. Because $\alpha + \beta \leq 1$, $\beta \in (1 - \varepsilon, 1)$ implies that $\alpha \in (0, \varepsilon)$, so the above also holds if β is sufficiently large.

Finally, $x^d < x^{**}$ follows from the first-order conditions $q_x((1+\gamma)\tau^d, x^d) f(\tau^d) = 1$ and $q_x((1+\gamma)\tau^{**}, x^{**})/\delta = 1$, $f(\tau^d) < 1/\delta$, $q_{xx} < 0$ and $q_{\tau x} \geq 0$. ■

Proof of Proposition 6: Imposing a fee-upon-sale is equivalent to increasing c for both the monopolist and the duopolists. Imposing a fee-upon-disposal is equivalent to increasing c for the monopolist because consumers dispose of the monopolist's last-generation product when they buy the monopolist's new product. Thus, the monopolist incurs the fee simultaneously with introducing a new product, and can defer the fee by extending the development time. However, imposing a fee-upon-disposal is not equivalent to increasing c in the duopoly model, because a duopolist incurs the fee when its competitor introduces a new product. In the unique duopoly equilibrium, each firm i sets (τ_i, x_i) to maximize (15), in the belief that the competitor is committed to introduce a new product in $\tau^d_{-i} = \tau^d$ units of time. Firm i knows that its choice of (τ_i, x_i) cannot affect the time until the competitor's next new product introduction, and hence cannot affect the time at which it incurs the fee. Therefore (τ^d, x^d) remains, unchanged, the unique duopoly equilibrium under a fee-upon-disposal.

To complete the proof, we must show that τ^m, x^m, $q(\tau^m, x^m)$, τ^d, x^d, and $q((1+\gamma)\tau^d, x^d)$ all strictly increase with c. Referring to $H_m(\tau)$ as defined in (EC.18), τ^m is the unique solution to $H_m(\tau) = 0$ and $\partial H_m/\partial\tau < 0$, so

$$\frac{\partial \tau^m}{\partial c} = -\frac{\partial H_m/\partial\tau}{\partial H_m/\partial c} = -\frac{\partial H_m/\partial\tau}{f(\tau_m)} > 0.$$

Given τ^m, x^m is the unique solution to the first-order condition $q_x(\tau^m, x^m)f(\tau^m) = 1$, and therefore

$$\frac{\partial x^m}{\partial c} = \frac{dx^m}{d\tau^m}\frac{\partial \tau^m}{\partial c} = -\frac{q_{\tau x} f + q_x f_\tau}{q_{xx} f}\frac{\partial \tau^m}{\partial c} > 0.$$

The equilibrium incremental quality per new product $q(\tau^m, x^m)$ also strictly increases with c because it is a strictly increasing function of both τ^m and x^m. By applying the same arguments with $H_d(\tau)$ as defined in (EC.38) substituting for $H_m(\tau)$, one may conclude that τ^d, x^d, and $q((1+\gamma)\tau^d, x^d)$ all strictly increase with c. ■

Proof of Proposition 7: As explained in detail in the proof of Proposition 6, imposing a fee-upon-sale or a fee-upon-disposal is equivalent to increasing c for the monopolist. Hence we must show that the monopolist's equilibrium profit

$$\pi^m(\tau^m, x^m) = \frac{e^{-\delta\tau^m}}{1 - e^{-\delta\tau^m}}[q(\tau^m, x^m)f(\tau^m) - x^m - c]$$

strictly increases with c if and only if (19) is satisfied. Because

$$\frac{\partial \pi^m}{\partial x^m} = \frac{e^{-\delta\tau^m}}{1 - e^{-\delta\tau^m}}[q_x(\tau^m, x^m)f(\tau^m) - 1] = 0, \tag{EC.41}$$

where the second equality comes from (EC.5) in Lemma 1,

$$\begin{aligned}
\frac{d\pi^m}{dc} &= \frac{\partial \pi^m}{\partial \tau^m}\frac{\partial \tau^m}{\partial c} + \frac{\partial \pi^m}{\partial x^m}\frac{\partial x^m}{\partial c} - \frac{e^{-\delta\tau^m}}{1 - e^{-\delta\tau^m}} \\
&= \frac{\partial \pi^m}{\partial \tau^m}\frac{\partial \tau^m}{\partial c} - \frac{e^{-\delta\tau^m}}{1 - e^{-\delta\tau^m}} \\
&= \left[\frac{-\delta e^{-\delta\tau^m}}{(1 - e^{-\delta\tau^m})^2}(qf - x^m - c) + \frac{e^{-\delta\tau^m}}{1 - e^{-\delta\tau^m}}(q_\tau f + q f_\tau)\right]\frac{\partial \tau^m}{\partial c} - \frac{e^{-\delta\tau^m}}{1 - e^{-\delta\tau^m}}
\end{aligned}$$

$$= \frac{e^{-\delta\tau^m}}{1-e^{-\delta\tau^m}}\left[(-q + \frac{(x^m+c)}{f} + q_\tau f + q f_\tau)\frac{\partial \tau^m}{\partial c} - 1\right]$$
$$= \frac{e^{-\delta\tau^m}}{1-e^{-\delta\tau^m}}\left(q f_\tau \frac{\partial \tau^m}{\partial c} - 1\right),$$

where the final equality follows from (EC.4) in Lemma 1. It follows that the monopolist's profit strictly increases with c if and only if

$$q f_\tau - \frac{1}{\partial\tau^m/\partial c} > 0. \tag{EC.42}$$

(We established in Proposition 6 that $\partial\tau^m/\partial c > 0$.) Using the implicit function theorem, differentiating

$$c = qf\left(1 - \beta\frac{f}{\tau^m} - \alpha\right)$$

in (EC.2),

$$\frac{1}{\partial\tau^m/\partial c} = \left(q f_\tau + q_\tau f + q_x f \frac{dx}{d\tau}\right)\left(1 - \alpha - \frac{\beta}{\tau^m} f\right) + q\frac{\beta f}{\tau^m}\left(\frac{f}{\tau^m} - f_\tau\right)$$

Applying (EC.3) for $dx^m/d\tau^m$, the above equals

$$\frac{(q f_\tau + q_\tau f)}{1-\alpha}\left(1 - \alpha - \frac{\beta}{\tau^m} f\right) + q\frac{\beta f}{\tau^m}\left(\frac{f}{\tau^m} - f_\tau\right)$$
$$= q f_\tau\left(1 - \frac{\beta f}{(1-\alpha)\tau^m}\right) + q_\tau f\left[1 + \frac{f}{\tau^m}\left(1 - \frac{\beta}{1-\alpha}\right) - f_\tau\right]$$
$$= q f_\tau + q_\tau f\left[1 + \frac{f}{\tau^m}\left(1 - \frac{\beta}{1-\alpha}\right) - \frac{2-\alpha}{1-\alpha} f_\tau\right].$$

Substituting the above expression for $\frac{1}{\partial\tau^m/\partial c}$ into (EC.42), we find that the monopolist's profit strictly increases with c if and only if

$$\left(1 - \frac{\beta}{1-\alpha}\right) < \frac{\tau^m}{f}\left(\frac{2-\alpha}{1-\alpha} f_\tau - 1\right). \tag{EC.43}$$

The above is true when $\tau^m = 0$, false when $\tau^m = \infty$, and the derivative of the right-hand side

$$\left(\frac{1}{f} - \frac{\tau^m}{f^2} f_\tau\right)\left(\frac{2-\alpha}{1-\alpha} f_\tau - 1\right) - \frac{2-\alpha}{1-\alpha}\frac{\tau^m}{f}\delta f_\tau$$

$$
\begin{aligned}
&= \frac{2-\alpha}{1-\alpha}\frac{f_\tau}{f} - \frac{1}{f} + \frac{\tau^m}{f^2} f_\tau \left(1 - \frac{2-\alpha}{1-\alpha}(f_\tau + \delta f)\right) \\
&= \frac{2-\alpha}{1-\alpha}\frac{f_\tau}{f} - \frac{1}{f} + \frac{\tau^m}{f^2} f_\tau \left(1 - \frac{2-\alpha}{1-\alpha}\right) \\
&= \left(\frac{f_\tau}{f} - \frac{1}{f}\right) + \frac{f_\tau}{(1-\alpha)f}\left(1 - \frac{\tau^m}{f}\right) \\
&< 0
\end{aligned}
$$

so there exists a unique $\tau^m_{profit} > 0$ at which the two sides of (EC.43) are equal and (EC.43) holds at τ^m if and only if $\tau^m < \tau^m_{profit}$. As $\partial \tau^m / \partial c > 0$, there exists some $c^m_{profit} > 0$ such that $\tau^m < \tau^m_{profit}$, and thus $d\pi^m/dc > 0$ if and only if $c < c^m_{profit}$.

Analogous arguments apply for the duopoly model. As explained in detail in the proof of Proposition 6, imposing a fee-upon-sale is equivalent to increasing c for duopolists. Therefore we must show that the duopolists' profit

$$
\pi^d = \frac{e^{-\delta\tau^d}}{1-e^{-\delta\tau^d}}[q((1+\gamma)\tau^d, x^d) f(\tau^d) - x^d - c]
$$

increases with c if and only if (20) is satisfied. The duopolist's first-order condition for expenditure (EC.10) implies that

$$
\frac{\partial \pi^d}{\partial x^d} = \frac{e^{-\delta\tau^d}}{1-e^{-\delta\tau^d}}[q_x((1+\gamma)\tau^d, x^d) f(\tau^d) - 1] = 0.
$$

Therefore,

$$
\begin{aligned}
\frac{d\pi^d}{dc} &= \frac{\partial \pi^d}{\partial \tau^d}\frac{\partial \tau^d}{\partial c} + \frac{\partial \pi^d}{\partial x^d}\frac{\partial x^d}{\partial c} - \frac{e^{-\delta\tau^d}}{1-e^{-\delta\tau^d}} \\
&= \frac{\partial \pi^d}{\partial \tau^d}\frac{\partial \tau^d}{\partial c} - \frac{e^{-\delta\tau^d}}{1-e^{-\delta\tau^d}} \\
&= \left[\frac{-\delta e^{-\delta\tau^d}}{(1-e^{-\delta\tau^d})^2}(qf - x^d - c) + \frac{e^{-\delta\tau^d}}{1-e^{-\delta\tau^d}}((1+\gamma)q_\tau f + q f_\tau)\right]\frac{\partial \tau^d}{\partial c} - \frac{e^{-\delta\tau^d}}{1-e^{-\delta\tau^d}} \\
&= \frac{e^{-\delta\tau^d}}{(1-e^{-\delta\tau^d})}\left\{\left[\frac{-q_\tau f + q f_\tau}{1-e^{-\delta\tau^d}} + (1+\gamma)q_\tau f + q f_\tau\right]\frac{\partial \tau^d}{\partial c} - 1\right\} \\
&= \frac{e^{-\delta\tau^d}}{(1-e^{-\delta\tau^d})}\left\{\frac{[\gamma - (1+\gamma)e^{-\delta\tau^d}]q_\tau f + (2-f_\tau) q f_\tau}{1-e^{-\delta\tau^d}}\frac{\partial \tau^d}{\partial c} - 1\right\}
\end{aligned}
$$

where the next-to-last equality follows from the duopolist's first-order condition for τ^d (EC.9) and $\delta f = 1 - f_\tau = 1 - e^{-\delta\tau^d}$. It follows that the duopolists' profit strictly increases with c if and only if

$$
[\gamma - (1+\gamma)e^{-\delta\tau^d}]q_\tau f + (2-f_\tau) q f_\tau > \frac{(1-e^{-\delta\tau^d})}{\partial \tau_d / \partial c}. \tag{EC.44}
$$

(We know from Proposition 6 that $\partial\tau_d/\partial c > 0.$). From (EC.8)

$$\left(\frac{\partial q}{\partial \tau^d} + q_x\frac{dx}{d\tau}\right) = \beta\frac{q}{\tau^d} + \frac{1}{f}\left(\frac{(\partial q/\partial\tau^d)f + qf_\tau}{-q_{xx}}\right) = \frac{\beta}{(1-\alpha)\tau^d}q + \frac{\alpha}{(1-\alpha)f}qf_\tau$$

From (EC.7) in Lemma 2,

$$\begin{aligned}
\frac{1}{\partial\tau^d/\partial c} &= \left(\frac{\partial q}{\partial\tau^d} + q_x\frac{dx}{d\tau}\right)\left[\frac{1}{\delta} - \frac{\beta}{\delta}\frac{f}{(1+\gamma)\tau^d} - \alpha f\right] \\
&\quad + \left[\frac{\beta}{\delta}\frac{f}{(1+\gamma)(\tau^d)^2} - \frac{\beta}{\delta}\frac{f_\tau}{(1+\gamma)\tau^d} - \alpha f_\tau\right]q \\
&= \frac{q}{1-\alpha}\left(\frac{\beta}{\tau^d} + \alpha\frac{f_\tau}{f}\right)\left[\frac{1}{\delta} - \frac{\beta}{\delta}\frac{f}{(1+\gamma)\tau^d} - \alpha f\right] \\
&\quad + \left[\frac{\beta}{\delta}\frac{f}{(1+\gamma)(\tau^d)^2} - \frac{\beta}{\delta}\frac{f_\tau}{(1+\gamma)\tau^d} - \alpha f_\tau\right]q \\
&= \left(\frac{1}{\delta f} - \alpha - (1-\alpha)\right)\frac{\alpha}{1-\alpha}qf_\tau + \frac{\beta f}{(1+\gamma)\delta(\tau^d)^2}\left(1 - \frac{\beta}{1-\alpha}\right)q \\
&\quad + \frac{\beta}{\delta\tau^d}\left(\frac{1}{1-\alpha} - \frac{\alpha\delta f}{1-\alpha} - \frac{\alpha f_\tau}{(1-\alpha)(1+\gamma)} - \frac{f_\tau}{(1+\gamma)}\right)q \\
&= \frac{\alpha f_\tau^2}{(1-\alpha)\delta f}q + \frac{\beta\delta f}{(1+\gamma)(\delta\tau^d)^2}\left(1 - \frac{\beta}{1-\alpha}\right)q \\
&\quad + \frac{\beta}{(1+\gamma)\delta\tau^d}\left(1 + \gamma - f_\tau + \frac{\alpha\gamma}{1-\alpha}f_\tau\right)q
\end{aligned}$$

By substituting the right hand side above for $\frac{1}{\partial\tau^d/\partial c}$, (EC.44) is equivalent to

$$\begin{aligned}
&\beta[\gamma - (1+\gamma)e^{-\delta\tau^d}]\frac{1 - e^{-\delta\tau^d}}{(1+\gamma)\delta\tau^d} + (2 - f_\tau)f_\tau \\
&> \frac{\alpha f_\tau^2}{(1-\alpha)} + \frac{\beta(1 - e^{-\delta\tau^d})^2}{(1+\gamma)(\delta\tau^d)^2}\left(1 - \frac{\beta}{1-\alpha}\right) + \frac{\beta(1 - e^{-\delta\tau^d})}{(1+\gamma)\delta\tau^d}\left(1 + \gamma - f_\tau + \frac{\alpha\gamma}{1-\alpha}f_\tau\right).
\end{aligned}$$

By denoting $z = \delta\tau^d$, dividing both sides by f_τ, and rearranging terms, the above inequality simplifies into

$$(2 - \frac{e^{-z}}{1-\alpha}) - \frac{\beta(1-e^{-z})^2}{(1+\gamma)z^2}\left(1 - \frac{\beta}{1-\alpha}\right)e^z - \frac{\beta(1-e^{-z})}{(1+\gamma)z}\left(e^z + \frac{\gamma}{1-\alpha}\right) > 0. \tag{EC.45}$$

Let $g(z)$ denote the left hand side of (EC.45). The duopolists are profitable at the equilibrium and therefore, from (EC.25), that for firms to be profitable at the equilibrium,

$$\tau^d \geq \frac{z_0}{\delta} \text{ where } \frac{(1+\gamma)z_0}{e^{z_0} - 1} = \beta.$$

It is easy to verify that $g(\infty) < 0$ and

$$\begin{aligned} g(z_0) &= \left(2 - \frac{e^{-z_0}}{1-\alpha}\right) - \frac{\beta(1-e^{-z_0})^2}{(1+\gamma)z_0^2}\left(1 - \frac{\beta}{1-\alpha}\right)e^{z_0} - \frac{\beta(1-e^{-z_0})}{(1+\gamma)z_0}\left(e^{z_0} + \frac{\gamma}{1-\alpha}\right) \\ &= 2 - \frac{e^{-z_0}}{1-\alpha} - \frac{1-e^{-z_0}}{z_0}\left(1 - \frac{(1+\gamma)z_0}{(1-\alpha)(e^{z_0}-1)}\right) - \left(1 + \frac{\gamma}{1-\alpha}e^{-z_0}\right) \\ &= 1 - \frac{1-e^{-z_0}}{z_0} \\ &> 0. \end{aligned}$$

We will prove that there exists a unique point $z^d_{profit} \in (z_0, \infty)$ such that $g(z^d_{profit}) = 0$ and $g(z) > (<)0$ for $z < (>)\ z^d_{profit}$. (Prior arguments show that this will imply that the duopolists' profit strictly increases with c if and only if $\tau^d < z^d_{profit}/\delta$.) It will be sufficient to verify that $g''(z) < 0$ which is true because in (EC.45), components

$$\frac{e^{-z}}{1-\alpha}, \frac{(1-e^{-z})^2}{z^2}e^z, \frac{e^z-1}{z}, \text{ and } \frac{1-e^{-z}}{z},$$

are all convex functions of z.

We conclude that the duopolists' profit strictly increases with c if and only if

$$\tau^d < z^d_{profit}/\delta. \tag{EC.46}$$

Because τ^d strictly increases with c by Proposition 6, (EC.46) is equivalent to

$$c < c^d_{profit} = C(z^d_{profit})$$

where $C^d(z)$ is defined by (EC.7) using $\tau = z/\delta$.

Observe that $C^d(z)$ strictly decreases with δ. As z^d_{profit} is invariant with respect to δ, this establishes that c^d_{profit} strictly decreases with δ.

Now suppose $\alpha+\beta = 1$, in which case (EC.43) can be simplified to $\delta\tau^m < z^m_{profit} = \ln[1+1/(1-\alpha)]$, and the left-hand side of (EC.45), $g(z)$, becomes

$$g(z) = \left(2 - \frac{e^{-z}}{1-\alpha}\right) - \frac{(1-e^{-z})}{(1+\gamma)z}[(1-\alpha)e^z + \gamma]$$

Because

$$\begin{aligned} g(z^m_{profit}) &= g(\ln[1+1/(1-\alpha)]) \\ &= 2 - \frac{1}{2-\alpha} - \frac{[1+\gamma/(2-\alpha)]/(1+\gamma)}{\ln[1+1/(1-\alpha)]} \\ &= \frac{(3-2\alpha)\ln[1+1/(1-\alpha)] - 1 - (1-\alpha)/(1+\gamma)}{(2-\alpha)\ln[1+1/(1-\alpha)]} \\ &\geq \frac{(3-2\alpha)\ln[1+1/(1-\alpha)] - 2 + \alpha}{(2-\alpha)\ln[1+1/(1-\alpha)]} \\ &> 0, \end{aligned}$$

and $g(z) \leq 0$ for $z \geq z^d_{profit}$, $z^m_{profit} < z^d_{profit}$. As we have shown previously, τ^d increases in c, so $C^d(z)$ increases in z and thus $C^d(z^d_{profit}) > C^d(z^m_{profit})$. To show $c^m_{profit} < c^d_{profit}$, it is sufficient to prove that $c^m_{profit} < C^d(z^m_{profit})$. For brevity denote z^m_{profit} by $\overline{z}$, from (EC.7), when $\beta = 1-\alpha$,

$$\begin{aligned} C^d(z) &= \frac{[\alpha(1-e^{-\overline{z}})]^{\alpha/(1-\alpha)}[(1+\gamma)\overline{z}]}{\delta^{(2-\alpha)/(1+\alpha)}} \left[1 - \alpha(1-e^{-\overline{z}}) - (1-\alpha)\frac{1-e^{-\overline{z}}}{\overline{z}}\right] \\ &\geq \frac{[\alpha(1-e^{-\overline{z}})]^{1/(1-\alpha)}}{\alpha\delta^{(2-\alpha)/(1+\alpha)}} \left[\frac{\overline{z}}{1-e^{-\overline{z}}} - \alpha\overline{z} - (1-\alpha)\right] \\ &> \frac{[\alpha(1-e^{-\overline{z}})]^{1/(1-\alpha)}}{\alpha\delta^{(2-\alpha)/(1+\alpha)}} [e^{-\overline{z}} + \overline{z} - \alpha\overline{z} - (1-\alpha)] \\ &\geq \frac{[\alpha(1-e^{-\overline{z}})]^{1/(1-\alpha)}}{\alpha\delta^{(2-\alpha)/(1+\alpha)}} (1-\alpha)[\overline{z} - (1-e^{-\overline{z}})] \\ &= c^m_{profit} \quad \text{by using (EC.2) with } \beta = 1-\alpha \text{ and } \tau^m = \tau^m_{profit} = \overline{z}/\delta. \quad \blacksquare \end{aligned}$$

Proof of Proposition 8: Social welfare at the duopoly equilibrium is

$$W^d = \frac{e^{-\delta\tau^d}}{1-e^{-\delta\tau^d}} \left[\frac{q((1+\gamma)\tau^d, x^d)}{\delta} - x - c - e^{-\delta\tau^d}(k+z)\right];$$

note that a fee-upon-sale is a transfer from the duopolists to other sectors of the economy (collection and recycling), so it does not directly affect social welfare. Proposition 6 established that imposing a fee-upon-sale strictly increases τ^d. At the duopoly equilibrium,

$$\begin{aligned} \frac{dW^d}{d\tau^d} &= \frac{-\delta e^{-\delta\tau^d}}{(1-e^{-\delta\tau^d})^2} \left(\frac{q}{\delta} - x^d - c - (2e^{-\delta\tau^d} - e^{-2\delta\tau^d})(k+z)\right) \\ &+ \frac{e^{-\delta\tau^d}}{1-e^{-\delta\tau^d}} \left[\frac{(1+\gamma)q_\tau}{\delta} + (\frac{q_x}{\delta} - 1)\frac{dx^d}{d\tau^d}\right] \\ &= \frac{e^{-\delta\tau^d}}{(1-e^{-\delta\tau^d})^2} \left[-q + \delta(x^d + c) + \delta e^{-\delta\tau^d}(2-e^{-\delta\tau^d})(k+z) + (1+\gamma)q_\tau f + f_\tau \frac{dx^d}{d\tau^d}\right] \end{aligned}$$

$$= \frac{e^{-\delta\tau^d}}{(1-e^{-\delta\tau^d})^2}\left(\delta e^{-\delta\tau^d}(2-e^{-\delta\tau^d})(k+z)+\gamma q_\tau f + f_\tau \frac{dx^d}{d\tau^d}\right) \qquad \text{because of } (EC.28)$$
$$> 0. \qquad \text{(EC.47)}$$

Hence imposing a fee-upon-sale strictly increases social welfare in the duopoly model.

Social welfare at the monopoly equilibrium is

$$W^m = \frac{e^{-\delta\tau^m}}{1-e^{-\delta\tau^m}}\left[\frac{q(\tau^m,x^m)}{\delta} - x^m - c - e^{-\delta\tau^m}(k+z)\right]; \qquad \text{(EC.48)}$$

note that a fee-upon-sale or a fee-upon-disposal is a transfer from the monopolist to other sectors of the economy (collection and recycling), and does not directly affect social welfare. In the monopoly model, imposing a fee-upon-sale or fee-upon-disposal is equivalent to increasing c. Proposition 6 tells us that imposing either a fee-upon-sale or a fee-upon-disposal will strictly increase τ^m. Therefore imposing a small fee-upon-sale or fee-upon-disposal strictly increases social welfare if the following quantity is positive

$$\begin{aligned}
&\frac{\partial W^m}{\partial \tau^m}\frac{\partial \tau^m}{\partial c} + \frac{\partial W^m}{\partial x^m}\frac{\partial x^m}{\partial c} \\
&= \frac{e^{-\delta\tau^m}}{(1-e^{-\delta\tau^m})^2}[(-q+\delta(x^m+c)+q_\tau f)+\delta e^{-\delta\tau^m}(2-e^{-\delta\tau^m})(k+z) \\
&\qquad\qquad + \left(\frac{q_x}{\delta}-1\right)(1-e^{-\delta\tau^m})]\frac{dx^m}{d\tau^m}\frac{\partial \tau^m}{\partial c} \\
&= \frac{e^{-\delta\tau^m}}{(1-e^{-\delta\tau^m})^2}\left(q_\tau f - q + \frac{dx^m}{d\tau^m} + \delta e^{-\delta\tau^m}(2-e^{-\delta\tau^m})(k+z)\right) f_\tau \frac{\partial \tau^m}{\partial c},
\end{aligned}$$

where the final equality follows from (EC.4), (EC.5), and $\delta f + f_\tau = 1$. Assuming a Cobb-Douglas quality function, the expression above is strictly positive if and only if

$$q - q_\tau f < \frac{dx^m}{d\tau^m} + \delta e^{-\delta\tau^d}(2-e^{-\delta\tau^d})(k+z) = \frac{\alpha}{1-\alpha}(qf_\tau + q_\tau f) + \delta e^{-\delta\tau^m}(2-e^{-\delta\tau^m})(k+z), \quad \text{(EC.49)}$$

where the equality follows from (EC.3) in Lemma 1. Note that τ^m is invariant with respect to $(k+z)$, and the right-hand side of (EC.49) strictly increases in $(k+z)$ and converges to ∞ as $(k+z)\to\infty$. Hence there exists a finite constant $\bar{k}$, such that a fee-upon-sale or fee-upon-disposal strictly increases social welfare if $(k+z) > \bar{k}$ but strictly decreases social welfare if $(k+z) < \bar{k}$.

Furthermore, the constant $\bar{k}$ is strictly positive if the inequality in (EC.49) is violated at $k+z=0$, that is, if

$$q - q_\tau f \geq \frac{\alpha}{1-\alpha}(qf_\tau + q_\tau f),$$

which is true if and only if (using $q_\tau = \beta q/\tau^m$ in the above and with a simple transformation),

$$\tau^m \geq \frac{\varpi^m_{welfare}}{\delta} \tag{EC.50}$$

where $\varpi^m_{welfare}$ is implicitly defined by

$$(1-e^{-\varpi})\frac{\beta}{\varpi} + \alpha(1+e^{-\varpi}) = 1. \tag{EC.51}$$

Because τ^m strictly increases with c by Proposition 6, (EC.50) is equivalent to

$$c \geq c^m_{welfare} = \frac{\alpha^{\alpha/(1-\alpha)}}{\delta^{1/(1-\alpha)+1}}(1-e^{-\varpi^m_{welfare}})^{1/(1-\alpha)}\left(1-\alpha-\beta\frac{1-e^{-\varpi^m_{welfare}}}{\varpi^m_{welfare}}\right)\varpi^m_{welfare},$$

where $c^m_{welfare}$ is defined by (EC.2) with $\varpi^m_{welfare} = \delta\tau^m_{welfare}$.

We now prove the corollary, stated in the paper immediately after Proposition 8, that $c < c^m_{welfare}$ implies that $x^m > c$. From (EC.49),

$$q_\tau f \geq q(1-\alpha-\alpha f_\tau).$$

From (EC.2)

$$c = qf\left(1-\alpha-\beta\frac{f}{\tau^m}\right) = (1-\alpha)qf - q_\tau f^2.$$

By canceling out $q_\tau f$ in the above,

$$c \leq (1-\alpha)qf - q(1-\alpha-\alpha f_\tau)f = \alpha q f f_\tau = x^m q_x f f_\tau = x^m f_\tau \leq x^m.$$

Finally, note that our definition (9)-(23) of social welfare does not include the environmental, health and end-of-life processing costs for the "generation zero" product that is introduced at time zero and disposed at time τ. (This is consistent with the fact that we do not include the R&D and production cost for the generation zero product in calculating manufacturers' profits and social welfare, and the investment in design for recyclability to reduce k, incorporated in Section 6, does not affect the generation zero product.) A fee-upon-sale postpones the disposal of the generation zero product and so reduces its discounted environmental, health and processing cost, as does a fee-upon-disposal in the monopoly model. ■

Proof of Proposition 9: In the duopoly model, consumer surplus is

$$CS^d = \frac{e^{-\delta\tau^d}}{1-e^{-\delta\tau^d}}\left(\frac{q((1+\gamma)\tau^d, x^d)}{\delta} - q((1+\gamma)\tau^d, x^d)f(\tau^d)\right) = \frac{e^{-2\delta\tau^d}}{\delta(1-e^{-\delta\tau^d})}q((1+\gamma)\tau^d, x^d). \tag{EC.52}$$

Proposition 6 shows that imposing a fee-upon-disposal has no effect on the duopoly equilibrium (τ^d, x^d), and therefore the fee-upon-disposal does not affect consumer surplus. Proposition 6 also shows that imposing a fee-upon-sale strictly increases τ^d, so for a small fee-upon-sale to increase consumer surplus, it is necessary that

$$\frac{dCS^d}{d\tau^d} = \frac{e^{-2\delta\tau^d}}{\delta(1-e^{-\delta\tau^d})}\left[\frac{\partial q}{\partial\tau^d} + q_x\frac{dx^d}{d\tau^d} - \delta q\left(\frac{2-e^{-\delta\tau^d}}{1-e^{-\delta\tau^d}}\right)\right] \geq 0$$

which is true if and only if (note $q_x f = 1$ and $f = (1-e^{-\delta\tau})/\delta$),

$$(2-e^{-\delta\tau^d})q - \frac{\partial q}{\partial\tau^d}f \leq \frac{dx^d}{d\tau^d} = \frac{\alpha}{1-\alpha}\left(\frac{\partial q}{\partial\tau^d}f + qf_\tau\right)$$

where the last equality comes from (EC.8). By substituting $\partial q/\partial\tau^d = \beta q/\tau^d$ and canceling q from both sides,

$$\left(2 - \frac{f_\tau}{1-\alpha}\right) \leq \frac{\beta}{(1-\alpha)}\frac{f}{\tau^d}.$$

Since $f < \tau^d$ and $\beta \leq 1-\alpha$, the above inequality requires $f_\tau > 1-\alpha$, which cannot be true because the first-order condition (EC.9) implies that

$$q_\tau f - qf_\tau = \delta(qf - x^d - c).$$

To enable the duopolists to earn a profit, $qf - x^d - c > 0$ and therefore

$$f_\tau < \frac{q_\tau f}{q} = \frac{f}{(1+\gamma)\tau^d}\beta \leq \frac{f}{\tau^d}\beta < \beta \leq 1-\alpha.$$

In the monopoly model, consumer surplus is

$$CS^m = \frac{e^{-\delta\tau^m}}{1-e^{-\delta\tau^m}}\left(\frac{q(\tau^m,x^m)}{\delta} - q(\tau^m,x^m)f(\tau^m)\right) = \frac{e^{-2\delta\tau^m}}{\delta(1-e^{-\delta\tau^m})}q(\tau^m,x^m). \tag{EC.53}$$

A slight increase of equilibrium development time τ^m changes consumer surplus by

$$\frac{dCS^m}{d\tau^m} = \frac{e^{-2\delta\tau^m}}{\delta(1-e^{-\delta\tau^m})}\left[q_\tau + q_x\frac{dx^m}{d\tau^m} - \delta q\left(\frac{2-e^{-\delta\tau^m}}{1-e^{-\delta\tau^m}}\right)\right]. \tag{EC.54}$$

Proposition 6 shows imposing a fee-upon-sale or fee-upon-disposal strictly increases τ^m, so consumer surplus strictly decreases if

$$q_\tau + q_x\frac{dx^m}{d\tau^m} - \delta q\left(\frac{2-e^{-\delta\tau^m}}{1-e^{-\delta\tau^m}}\right) < 0.$$

Using (EC.3) in Lemma 1 for $dx^m/d\tau^m$, the left-hand side of the above is

$$\begin{aligned} q_\tau + \frac{\alpha(q_\tau f + qf_\tau)}{(1-\alpha)f} - \delta q\frac{2-e^{-\delta\tau^m}}{1-e^{-\delta\tau^m}} &= q_\tau\frac{1}{1-\alpha} + \delta q\left(\frac{\alpha e^{-\delta\tau^m}}{(1-\alpha)(1-e^{-\delta\tau^m})} - \frac{2-e^{-\delta\tau^m}}{1-e^{-\delta\tau^m}}\right) \\ &= \frac{q_\tau}{1-\alpha} + \left[\frac{e^{-\delta\tau^m}-2(1-\alpha)}{(1-\alpha)(1-e^{-\delta\tau^m})}\right]\delta q. \end{aligned}$$

Because $q_\tau = \beta q/\tau^m$, the expression above is strictly negative if and only if

$$\beta\frac{(1-e^{-\delta\tau^m})}{\delta\tau^m} + e^{-\delta\tau^m} < 2(1-\alpha). \tag{EC.55}$$

Because its left-hand side strictly decreases in τ^m and τ^m strictly increases in c, there exists some c^m_{cs} such that (EC.55) is satisifed if and only if $c > c^m_{cs}$. To demonstrate (EC.55) is always true when $c > x^m$, apply Cobb-Douglas formulation and substitute q_τ with $\beta q/\tau^m$ and q_x with $\alpha q/x^m$ in the first-order conditions (EC.4) and (EC.5),

$$\beta\frac{f}{\tau^m} = 1 - \frac{x^m + c}{qf} < 1 - \frac{2x^m}{qf} = 1 - \frac{2x^m}{q}q_x = 1 - 2\alpha.$$

Insert the above into (EC.55)

$$\beta\frac{(1-e^{-\delta\tau^m})}{\delta\tau^m} + e^{-\delta\tau^m} < 1 - 2\alpha + e^{-\delta\tau^m} < 2(1-\alpha). \qquad \blacksquare$$

Proof of Proposition 10: Under collective or individual EPR, the monopolist, as the only firm in the market, bears the end-of-life cost for its own products. Therefore the monopolist chooses the end-of-life cost k to

$$\max_{k\in(-c,\overline{k}]} \left\{ -I(k) + L(\tau^m, x^m) - \frac{e^{-\delta\tau^m}}{1-e^{-\delta\tau^m}} k \right\},$$
$$\text{where } L(\tau^m, x^m) \equiv \frac{e^{-\delta\tau^m}}{1-e^{-\delta\tau^m}} [q(\tau^m, x^m) f(\tau^m) - x^m - c]$$

and (τ^m, x^m) is the unique monopoly equilibrium development time and expenditure corresponding to end-of-life cost k, characterized by the first-order conditions (EC.16) and (EC.17) with $(k+c)$ substituted for c. Our assumptions that $I(k)$ is strictly convex for $k \in (-c, \overline{k}]$ and satisfies $\lim_{k\downarrow -c} I'(k) = -\infty$ and $I'(\overline{k}) = 0$ guarantee existence of a unique interior solution $k^m \in (-c, \overline{k})$ at which

$$-I'(k^m) = \frac{e^{-\delta\tau^m}}{1-e^{-\delta\tau^m}} - \frac{dL}{dk} = \frac{e^{-\delta\tau^m}}{1-e^{-\delta\tau^m}} - \frac{dL}{d\tau^m}\frac{d\tau^m}{dk}. \tag{EC.56}$$

Since $dL/d\tau^m > 0$ by Proposition 2 and $d\tau^m/dk > 0$ by Proposition 6, $dL/dk > 0$. Holding τ^m constant at the level corresponding to the monopolist's optimal end-of-life cost k^m, define

$$k^* = \arg\min_{k\in(-c,\overline{k}]} \left\{ I(k) + \frac{e^{-\delta\tau^m}}{1-e^{-\delta\tau^m}} k \right\}.$$

Our assumptions that $I(k)$ is strictly convex for $k \in (-c, \overline{k}]$ and satisfies $\lim_{k\downarrow -c} I'(k) = -\infty$ and $I'(\overline{k}) = 0$ imply that

$$-I'(k^*) = \frac{e^{-\delta\tau^m}}{1-e^{-\delta\tau^m}}, \tag{EC.57}$$

and (EC.56) implies that

$$-I'(k^m) < \frac{e^{-\delta\tau^m}}{1-e^{-\delta\tau^m}}. \tag{EC.58}$$

As $I()$ is strictly convex, (EC.57) and (EC.58) imply that

$$k^m > k^*.$$

In both the monopoly and duopoly models, the fee-upon-sale ARF is invariant with respect to the end-of-life cost k and therefore the firms have no incentive whatsoever to invest in reducing the end-of-life cost below its maximum $k = \overline{k}$.

Individual EPR is a fee-upon-disposal, as is collective EPR with total cost allocated according to share of products disposed by consumers. Hence by Proposition 6, the duopoly equilibrium development time and expenditure are invariant with respect to the end-of-life cost. Under collective EPR with total cost allocated according to share of products disposed by consumers, each duopolist bears the end-of-life cost for its own product at the time that the competitor introduces a new product. Our assumption that consumers are homogeneous implies that all consumers dispose of the last-generation product upon purchasing the new product, and therefore no mixing of products occurs in the return stream. This makes collective EPR with total cost allocated according to share of products disposed by consumers precisely equivalent to individual EPR. Therefore each duopolist maximizes its profit by choosing the end-of-life cost to minimize the sum of investment and discounted end-of-life cost.

Under collective EPR with current-sales-based allocation, the end-of-life cost for a duopolist's product is paid by the competitor when the competitor introduces a new product and causes consumers to dispose of the duopolist's product. Therefore any investment by a duopolist to reduce the end-of-life cost of its product will only benefit the competitor by lowering its new product introduction cost. Moreover, the duopolist's profit declines with the end-of-life cost for its product because this effectively reduces its competitor's cost to introduce a new product and, by (15), causes the competitor to introduce new products more quickly. Therefore the duopolist has a dis-incentive to invest in reducing the end-of-life cost below its maximum $k = \overline{k}$. ■

Extending the Strategy Space to Allow Each Duopolist to Introduce Multiple Consecutive New Products: In the duopoly model formulation in §3, we restricted the strategy space so that each firm could introduce at most one new product in the expected time window between consecutive new product introductions by its competitor. That is, we assumed that the firms would alternate in new product introduction. We now assume $\gamma = 0$ and allow the duopolists to introduce new products at any time. In particular, a firm may introduce arbitrarily many new products in the expected time window between consecutive new product introductions by the competitor. The incremental quality for a new product is $q(\tau, x)$ where τ is the time elapsed since the last new product introduction by either firm and, as in Section 3, x is the development expenditure for that new product. Proposition EC.1 establishes that the unique stationary alternating equilibrium derived in Section 3 is a sequential equilibrium in the generalized state space. In the sequential equilibrium, each firm chooses to introduce exactly one new product between the consecutive new product introductions by its competitor, i.e., chooses to alternate with the competitor rather than introduce multiple new products in a row.

Proposition EC.1 *Suppose that $\gamma = 0$ and the duopolists may introduce new products at any time. The solution (τ^d, x^d) to (15) characterizes a sequential equilibrium in which the duopolists alternate in new product introduction, each duopolist chooses development time $2\tau^d$ and expenditure x^d for each new product, and the time between consecutive new product introductions is τ^d.*

Proof of Proposition EC.1: We will show that if a firm were to deviate from the alternating equilibrium (τ^d, x^d), then that firm would reduce its discounted profit and it would be sequentially rational for both firms to revert to the alternating equilibrium (τ^d, x^d) in the continuation game. Without loss of generality, we can assume that firm 1 introduces a new product at time 0, assume that firm 2 is committed to introduce a new product at time τ^d, and consider deviation by firm 1 from its equilibrium strategy during the time period $[0, 3\tau^d]$. (Recall that the equilibrium strategy has firm 1 introduce a new product at time $2\tau^d$ and firm 2 introduce a new product at time $3\tau^d$.) Consumers expect firm 2 to introduce a new product at time τ^d, so Lemma 3 implies that firm

1 will earn strictly negative profit on any new products introduced during $(0,\tau^d]$. By introducing new products during $(0,\tau^d]$, firm 1 cannot influence its own profit or the other firm's profit in the continuation game from time τ^d. Therefore introducing one or more products during $(0,\tau^d]$ can only strictly reduce firm 1's discounted profit.

Lemma 3 *There exists* $\tau_0 \in (\tau^d, 2\tau^d)$ *such that*

$$\max_{\tau\in[0,\bar{\tau}]\ x\geq 0}\{q(\tau,x)f(\bar{\tau}-\tau)-x-c\}\begin{Bmatrix} <0 \ for\ \bar{\tau}<\tau_0 \\ \geq 0\ for\ \bar{\tau}\geq\tau_0\end{Bmatrix} \tag{EC.59}$$

At time τ^d firm 2 introduces its new product and, following its equilibrium strategy, commits to introduce its next new product at time $3\tau^d$. Consumers expect firm 2 to introduce a new product at time $3\tau^d$. Therefore Lemma 3 implies that if firm 1 introduces a new product at any time $t \in (3\tau^d-\tau_0\ ,\ 3\tau^d)$, firm 1 will earn strictly negative profit on any additional new products that it introduces during $(t,3\tau^d]$. Proposition 4 established that if firm 1 introduces a single product during $[\tau^d,3\tau^d]$, it must do so at time $2\tau^d$ or earn lower discounted profit.

It remains to prove that firm 1 cannot introduce a product at time $t\in(\tau^d,3\tau^d-\tau_0]$, introduce second product during $(t,3\tau^d]$ and earn positive profit on both those two products. In proving this result, we can assume that firm 1 introduces no product during (τ^d,t). A product introduced during (τ^d,t) would earn strictly negative profit by Lemma 3 (for that product, $\bar{\tau}<t-\tau^d\leq 2\tau^d-\tau_0<\tau_0$) and would also strictly reduce the incremental quality and hence profit for the product introduced at time t. Let τ' and τ'' denote the optimal intervals for firm 1's two new products introduced during $(\tau^d,3\tau^d]$, so firm 1 introduces the first product at time $\tau^d+\tau'$ and the second one at time $\tau^d+\tau'+\tau''$ where $0<\tau'\leq 2\tau^d-\tau_0$ and $0<\tau''\leq 2\tau^d-\tau'$. With the corresponding optimal expenditures x' and x'', the incremental quality of the two products are $q(\tau',x')$ and $q(\tau'',x'')$ respectively. A sequential equilibrium requires consumers' beliefs to be consistent with the optimal actions of firm 1 after it deviates from the equilibrium. By Lemma 3, $\tau^d+\tau'\leq 3\tau^d-\tau_0<2\tau^d$, so the consumers see firm 1 deviate from its equilibrium strategy when it introduces the new product at time $\tau^d+\tau'$ and, from (EC.59), the consumers realize that firm 1 will optimally introduce another new product at time $\tau^d+\tau'+\tau''\leq 3\tau^d$. Therefore consumers are willing to pay $q(\tau',x')f(\tau'')$ for the first product and $q(\tau'', x'')f(\tau''')$ for the second product where $\tau''' = 2\tau^d - \tau' - \tau''$. Because x' and x'' maximize $q(\tau', x')f(\tau'') - x'$ and $q(\tau'', x'')f(\tau''') - x''$ respectively, the corresponding first-order conditions and properties of the Cobb-Douglas quality function (EC.1) imply that

$$q_x(\tau', x')f(\tau'') = \alpha(\frac{\tau'}{x'})^{1-\alpha} f(\tau'') = 1 \text{ and } q_x(\tau'', x'')f(\tau''') = \alpha(\frac{\tau''}{x''})^{1-\alpha} f(\tau''') = 1.$$

It follows that

$$x' = [\alpha f(\tau'')]^{\frac{1}{1-\alpha}} \tau', \quad q(\tau', x') = [\alpha f(\tau'')]^{\frac{\alpha}{1-\alpha}} \tau'$$
$$x'' = [\alpha f(\tau''')]^{\frac{1}{1-\alpha}} \tau'', \quad q(\tau'', x'') = [\alpha f(\tau''')]^{\frac{\alpha}{1-\alpha}} \tau''$$

so that

$$q(\tau', x')f(\tau'') - x' = (1-\alpha) q(\tau', x') f(\tau'') = (1-\alpha)\, \alpha^{\frac{\alpha}{1-\alpha}} [f(\tau'')]^{\frac{1}{1-\alpha}} \tau'$$
$$q(\tau'', x'')f(\tau''') - x'' = (1-\alpha) q(\tau'', x'') f(\tau''') = (1-\alpha)\, \alpha^{\frac{\alpha}{1-\alpha}} [f(\tau''')]^{\frac{1}{1-\alpha}} \tau''. \tag{EC.60}$$

From duopoly first-order condition (EC.7), with $\beta = 1-\alpha$ and $\gamma = 0$,

$$c > (1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}} [f(\tau^d)]^{\frac{1}{1-\alpha}} \tau^d \phi(\delta\tau^d),$$

where $\phi(\delta\tau^d) = 1/(1 - e^{-\delta\tau^d}) - 1/(\delta\tau^d)$, so for both new product introductions to be profitable,

$$q(\tau', x')f(\tau'') - x' > c > (1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}} [f(\tau^d)]^{\frac{1}{1-\alpha}} \tau^d \phi(\delta\tau^d)$$
$$q(\tau'', x'')f(\tau''') - x'' > c > (1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}} [f(\tau^d)]^{\frac{1}{1-\alpha}} \tau^d \phi(\delta\tau^d).$$

By (EC.60), this means

$$\min\{[f(\tau'')]^{\frac{1}{1-\alpha}} \tau' \,, [f(\tau''')]^{\frac{1}{1-\alpha}} \tau''\} > [f(\tau^d)]^{\frac{1}{1-\alpha}} \tau^d \phi(\delta\tau^d).$$

Define $y' = \delta\tau', y'' = \delta\tau'', y''' = \delta\tau'''$, and $y^d = \delta\tau^d$, the above inequality is equivalent to

$$\min\{(1 - e^{-y''})^{\frac{1}{1-\alpha}} y', (1 - e^{-y'''})^{\frac{1}{1-\alpha}} y''\} > (1 - e^{-y^d})^{\frac{1}{1-\alpha}} y^d \phi(y^d).$$

To prove that the two new products cannot both be profitable, it suffices to prove that the above cannot hold by showing for any y', y'', y''' such that $y' + y'' + y''' = 2y^d$,

$$\max_{(y',y'',y'''):y'+y''+y'''=2y^d}\{\min\{(1-e^{-y''})^{\frac{1}{1-\alpha}}y', (1-e^{-y'''})^{\frac{1}{1-\alpha}}y''\}\} < (1-e^{-y^d})^{\frac{1}{1-\alpha}}y^d\phi(y^d). \quad \text{(EC.61)}$$

The maximum on the left-hand side is always achieved with equality between the two terms within the min operator.

$$(1-e^{-y''})^{\frac{1}{1-\alpha}}y' = (1-e^{-y'''})^{\frac{1}{1-\alpha}}y''; \quad \text{(EC.62)}$$

to understand this, observe that if the two terms were not equal, one could increase the minimum of the two by increasing either y' or y''' and simultaneously reducing the other until both terms were equal. It follows that (EC.61) is true if and only if for all y', y'', and y''' that satisfy (EC.62),

$$(1-e^{-y''})^{\frac{1}{1-\alpha}}y' + (1-e^{-y'''})^{\frac{1}{1-\alpha}}y'' < 2(1-e^{-y^d})^{\frac{1}{1-\alpha}}y^d\phi(y^d) \quad \text{(EC.63)}$$

The first step to prove (EC.63) is to show that for all y', y'' and y''' satisfying (EC.62),

$$\frac{(1-e^{-y''})^{\frac{1}{1-\alpha}}y' + (1-e^{-y'''})^{\frac{1}{1-\alpha}}y''}{2(1-e^{-y^d})^{\frac{1}{1-\alpha}}y^d\phi(y^d)} \le \frac{(1-e^{-y''})y' + (1-e^{-y'''})y''}{2(1-e^{-y^d})y^d\phi(y^d)}, \quad \text{(EC.64)}$$

which is immediate if $y'' \le y^d$ and $y''' \le y^d$ so that

$$\left(\frac{1-e^{-y''}}{1-e^{-y^d}}\right)^{\frac{1}{1-\alpha}} \le \frac{1-e^{-y''}}{1-e^{-y^d}} \text{ and } \left(\frac{1-e^{-y'''}}{1-e^{-y^d}})^{\frac{1}{1-\alpha}}\right) \le \frac{1-e^{-y'''}}{1-e^{-y^d}}.$$

If this is not the case, then because $y' > 0$ and $y' + y'' + y''' = 2y^d$, at most one of y'' and y''' can be greater than y^d. Without loss of generality, let $y''' < y^d < y''$. Due to (EC.62), for all $0 \le \tilde{\alpha} \le \alpha$,

$$\begin{aligned}(1-e^{-y''})^{1/(1-\tilde{\alpha})}y' &= (1-e^{-y''})^{1/(1-\tilde{\alpha})}\frac{(1-e^{-y'''})^{\frac{1}{1-\alpha}}}{(1-e^{-y''})^{\frac{1}{1-\alpha}}}y'' \\ &= (1-e^{-y'''})^{1/(1-\tilde{\alpha})}y''(\frac{1-e^{-y'''}}{1-e^{-y''}})^{\frac{\alpha-\tilde{\alpha}}{(1-\alpha)(1-\tilde{\alpha})}} \\ &\le (1-e^{-y'''})^{1/(1-\tilde{\alpha})}y''. \end{aligned} \quad \text{(EC.65)}$$

Fix y', y'', and y''', and let

$$L(\tilde{\alpha}) = \frac{1}{2y^d\phi(y^d)}\left[y'\left(\frac{1-e^{-y''}}{1-e^{-y^d}}\right)^{\frac{1}{1-\tilde{\alpha}}} + y''\left(\frac{1-e^{-y'''}}{1-e^{-y^d}}\right)^{\frac{1}{1-\tilde{\alpha}}}\right]$$

For $0 \le \widetilde{\alpha} \le \alpha$,

$$\frac{dL(\widetilde{\alpha})}{d\widetilde{\alpha}} = \frac{1}{2y^d\phi(y^d)(1-\widetilde{\alpha})^2}\left[y'\left(\frac{1-e^{-y''}}{1-e^{-y^d}}\right)^{\frac{1}{1-\widetilde{\alpha}}}\ln\left(\frac{1-e^{-y''}}{1-e^{-y^d}}\right) + y''\left(\frac{1-e^{-y'''}}{1-e^{-y^d}}\right)^{\frac{1}{1-\widetilde{\alpha}}}\ln\left(\frac{1-e^{-y'''}}{1-e^{-y^d}}\right)\right]$$
$$\le \frac{y''}{2y^d\phi(y^d)(1-\widetilde{\alpha})^2}y''\left(\frac{1-e^{-y'''}}{1-e^{-y^d}}\right)^{\frac{1}{1-\widetilde{\alpha}}}\left[\ln\left(\frac{1-e^{-y''}}{1-e^{-y^d}}\right)+\ln\left(\frac{1-e^{-y'''}}{1-e^{-y^d}}\right)\right]$$

where the inequality follows from (EC.65). Because $y''+y'''=2y^d-y'<2y^d$,

$$\ln\left(\frac{1-e^{-y''}}{1-e^{-y^d}}\right)+\ln\left(\frac{1-e^{-y'''}}{1-e^{-y^d}}\right) \le \max_{0<y\le y''}\left\{\ln\left(\frac{1-e^{-y}}{1-e^{-y^d}}\right)+\ln\left(\frac{1-e^{-(2y^d-y)}}{1-e^{-y^d}}\right)\right\}<0,$$

so $dL(\widetilde{\alpha})/d\widetilde{\alpha}<0$ $(0\le\widetilde{\alpha}\le\alpha)$, which establishes that (EC.64) is true.

Under (EC.64), it suffices to prove (EC.63) by showing

$$\max_{y',\,y'',\,y'''}\{(1-e^{-y''})y'+(1-e^{-y'''})y''\}<2(1-e^{-y^d})y^d\phi(y^d). \tag{EC.66}$$

The first order conditions for the maximization problem on the left hand side of (EC.66) are

$$1-e^{-y''}=\lambda, \tag{EC.67}$$

$$y'e^{-y''}+(1-e^{-y'''})=\lambda, \tag{EC.68}$$

$$y''e^{-y'''}=\lambda \tag{EC.69}$$

where λ is the Lagrangian multiplier associated with the constraint $y'+y''+y'''=2y^d$. Eliminating λ from (EC.67) and (EC.69), $y'''=-\ln[(1-e^{-y''})/y'']$. Substituting that expression into (EC.68) and using (EC.67) to eliminate λ establishes that $y'=(e^{y''}-1)/y''-1$ and it follows that

$$(1-e^{-y''})y'+(1-e^{-y'''})y''=\frac{e^{y''}-2+e^{-y''}}{y''}+y''+2e^{-y''}-2.$$

Because $y^d=y'+y''+y'''$,

$$y^d=(y'+y''+y''')=0.5\left[\frac{e^{y''}-1}{y''}-1+y''-\ln\left(\frac{1-e^{-y''}}{y''}\right)\right]$$

and therefore

$$e^{-y^d}=\sqrt{\frac{1-e^{-y''}}{y''}}e^{\frac{1-y''+(1-e^{y''})/y''}{2}}$$

Recall $\phi(y^d) = 1/(1-e^{-y^d}) - 1/(y^d)$, so

$$\begin{aligned} 2(1-e^{-y^d})y^d\phi(y^d) &= 2y^d - 2 + 2e^{-y^d} \\ &= \frac{e^{y''}-1}{y''} - 1 + y'' - \ln\left(\frac{1-e^{-y''}}{y''}\right) - 2 + 2\sqrt{\frac{1-e^{-y''}}{y''}}e^{\frac{1-y''+(1-e^{y''})/y''}{2}}. \end{aligned}$$

Therefore, for $y'' > 0$,

$$\begin{aligned} &2(1-e^{-y^d})y^d\phi(y^d) - (1-e^{-y''})y' + (1-e^{-y'''})y'' \\ &= 2\sqrt{\frac{1-e^{-y''}}{y''}}e^{\frac{1-y''+(1-e^{y''})/y''}{2}} - 1 - \ln\left(\frac{1-e^{-y''}}{y''}\right) - 2e^{-y''} + \frac{1-e^{-y''}}{y''} \\ &> 0. \end{aligned} \tag{EC.70}$$

To verify the last inequality, notice that $-\ln[(1-e^{-y''})/y'']$ increases in y'', $1+2e^{-y''}$decreases in y'',and $-\ln[(1-e^{-8})/8] > 1+2e^{-8}$, so (EC.70) is strictly positive when $y'' \geq 8$. To verify that (EC.70) is strictly positive for $y'' \in (0,8)$, one may simply plot its value as a function of a single variable y'' over $[0,8]$. Therefore, (EC.66) is true, so (EC.63) and (EC.61) are true. This completes the proof that firm 1 cannot introduce a product at time $t \in (\tau^d, 3\tau^d - \tau_0]$, introduce second product during $(t, 3\tau^d]$ and earn positive profit on both those two products.

We conclude that by deviating from its equilibrium strategy during $[0, 3\tau^d]$, firm 1 can only reduce its discounted profit and cannot influence its own profit or firm 2's profit in the continuation game from time $3\tau^d$. By analogous arguments, it is sequentially rational for both firms to use their equilibrium strategies in the continuation game from time $3\tau^d$ even after a deviation by firm 1 during $[0, 3\tau^d]$. We conclude that the alternating equilibrium (τ^d, x^d) is a sequential equilibrium. ■

Proof of Lemma 3: Define

$$\tau_0 = \inf\{\overline{\tau} : \max_{\tau\in[0,\overline{\tau}]\ x\geq 0}\{q(\tau,x)f(\overline{\tau}-\tau) - x - c\} \geq 0\}$$

and observe that

$$0 < \tau_0 < 2\tau^d$$

because $q(0,x)=f(0)=0$ and the alternating equilibrium (τ^d,x^d) is profitable for the duopolists, so $q(\tau^d,x^d)f(\tau^d)-x^d-c>0$. As both $q(\tau,x)$ and $f(\tau)$ are continuous and strictly increasing functions of τ, for any $\overline{\tau}>\tau_0$,

$$\max_{\tau\in[0,\overline{\tau}]\ x\geq 0}\{q(\tau,x)f(\overline{\tau}-\tau)-x-c\}>\max_{\tau\in[0,\tau_0]\ x\geq 0}\{q(\tau,x)f(\tau_0-\tau)-x-c\}=0.$$

This completes the proof of (EC.59).

It remains to prove that $\tau_0>\tau^d$ by showing that

$$\max_{\tau\in[0,\tau^d]\ x\geq 0}\{q(\tau,x)f(\tau^d-\tau)-x-c\}<0. \tag{EC.71}$$

Let $(\widehat{\tau},\widehat{x})$ be an optimal solution to (EC.71), $\widehat{q}=q(\widehat{\tau},\widehat{x})$ and $\widehat{q}_\tau=q_\tau(\widehat{\tau},\widehat{x})$. The first-order condition

$$\widehat{q}_\tau f(\tau^d-\widehat{\tau})-\widehat{q}f_\tau(\tau^d-\widehat{\tau})=0$$

implies that

$$f(\tau^d-\widehat{\tau})=\frac{\widehat{q}}{\widehat{q}_\tau+\delta\widehat{q}},\text{ so }\delta\tau^d=\delta\widehat{\tau}+\ln\left(1+\frac{\delta\widehat{q}}{\widehat{q}_\tau}\right).$$

Substituting the expression for q_τ for Cobb-Douglas quality functions from (EC.1), the above equalities simplify to

$$\delta\tau^d=\delta\widehat{\tau}+\ln\left(1+\frac{\delta\widehat{\tau}}{1-\alpha}\right) \tag{EC.72}$$

$$\widehat{x}=\alpha q_x=[\alpha f(\tau^d-\widehat{\tau})]^{1/(1-\alpha)}\widehat{\tau}\quad\text{because } q_x(\widehat{\tau},\widehat{x})=\alpha\left(\frac{\widehat{\tau}}{\widehat{x}}\right)^{1-\alpha}=f(\tau^d-\widehat{\tau}), \tag{EC.73}$$

$$\text{and } \widehat{q}=[\alpha f(\tau^d-\widehat{\tau})]^{\alpha/(1-\alpha)}\widehat{\tau}, \tag{EC.74}$$

from (EC.16) and the properties (EC.1) of the Cobb-Douglas quality function. Since by (EC.73) and (EC.74),

$$q(\widehat{\tau},\widehat{x})f(\tau^d-\widehat{\tau})-\widehat{x}=(1-\alpha)\alpha^{\alpha/(1-\alpha)}[f(\tau^d-\widehat{\tau})]^{1/(1-\alpha)}\widehat{\tau}.$$

From (EC.7) with $\beta=1-\alpha$ and $\gamma=0$, and because $f(\tau^d-\widehat{\tau})<f(\tau^d)$, it is sufficient to prove $q(\widehat{\tau},\widehat{x})f(\tau^d-\widehat{\tau})-\widehat{x}<c$ by showing

$$\widehat{\tau}\leq\frac{\tau^d}{\delta}\left(\frac{1}{f(\tau^d)}-\frac{1}{\tau^d}\right)$$

or equivalently that

$$(\delta\widehat{\tau}+1)(1-e^{-\delta\tau^d}) \le \delta\tau^d.$$

To establish the above inequality, define $w=\delta\widehat{\tau}/(1-\alpha)$, so by (EC.72),

$$\begin{aligned}
\delta\tau^d &= \delta\widehat{\tau}+\ln\left(1+\frac{\delta\widehat{\tau}}{1-\alpha}\right)=(1-\alpha)w+\ln(1+w)\\
1-e^{-\delta\tau^d} &= 1-\frac{e^{-(1-\alpha)w}}{1+w}\\
\text{and}\quad (\delta\widehat{\tau}+1)(1-e^{-\delta\tau^d}) &= [(1-\alpha)w+1]\left(1-\frac{e^{-(1-\alpha)w}}{1+w}\right)\\
&= (1-\alpha)w+\left[1-e^{-(1-\alpha)w}\left(1-\frac{\alpha w}{1+w}\right)\right]
\end{aligned}$$

Because for any $w\ge 0$,

$$\frac{d\{e^{\alpha w}[1-\alpha w/(1+w)]\}}{d\alpha}=we^{\alpha w}\left(1-\frac{\alpha w+1}{1+w}\right)\ge 0$$

so $[1-e^{-(1-\alpha)w}(1-\frac{\alpha}{1+w}w)]$ is maximized at $\alpha=0$ when the value becomes $1-e^{-w}$. Thus

$$\begin{aligned}
(\delta\widehat{\tau}+1)(1-e^{-\delta\tau^d}) &= (1-\alpha)w+\left[1-e^{-(1-\alpha)w}\left(1-\frac{\alpha w}{1+w}\right)\right]\\
&\le (1-\alpha)w+1-e^{-w}\\
&< (1-\alpha)w+\ln(1+w)\\
&= \delta\tau^d,
\end{aligned}$$

which completes the proof. ■

Construction of the Numerical Examples in §5

In constructing the numerical examples in §5, we assume that the quality function is of the functional form $q(\tau, x) = vx^{\alpha}\tau^{\beta}$. Imposing a fee per unit sale ϕ increases non-R&D cost per new product introduction from c to $c + N\phi$ where N is the number of units sold per new product generation. We use publicly available data to fit the parameters v, α, β, c, and N, as described below.

In both monopoly and duopoly models, the first-order condition for equilibrium R&D expenditure is

$$q_x f = 1.$$

The Cobb-Douglas functional form of $q(\tau, x)$ implies that $xq_x = \alpha q$, so

$$\alpha = \frac{x}{qf}. \tag{EC.75}$$

The denominator qf is the revenue generated by a new product, and therefore α is the ratio of R&D expense to revenue, which we obtain from manufacturers' financial statements.

In the monopoly model, the first-order condition for equilibrium development time τ is

$$q_\tau f - q - (x + c)/f = 0.$$

Using the properties of $q(\tau, x)$ that $xq_x = \alpha q$ and $\tau q_\tau = \beta q$, the above equation is easily transformed into

$$\beta = \frac{\tau}{f(\tau)} \left(1 - \frac{x + c}{qf}\right) \tag{EC.76}$$

where the term

$$1 - \frac{(x + c)}{qf} \tag{EC.77}$$

is the operating margin for the new product. Thus we can compute β from public data on operating margins and the time between new product introductions (τ). We take an analogous approach in fitting the duopoly model. We transform the first-order condition for the equilibrium τ

$$q_\tau f - q - \delta(x + c) = 0$$

into

$$\beta = \frac{\tau}{f(\tau)}\left(1 - \frac{x+c}{qf}\delta f(\tau)\right). \tag{EC.78}$$

In all examples we considered, the Cobb-Douglas requirement that $\alpha+\beta \leq 1$ implied that $\gamma \approx 0$, and therefore we set $\gamma = 0$.

Straightforward manipulation of the functional form we assumed for the quality function gives

$$v = \frac{q}{x^{\alpha}\tau^{\beta}} = \frac{qf}{x^{\alpha}\tau^{\beta}f}, \tag{EC.79}$$

where the numerator qf is the revenue per new product, which we obtain from manufacturers' financial statements. Furthermore, having already obtained α, qf, β and τ, and recalling that $x = \alpha qf$ from (EC.75), we can then compute the value of v.Given revenue qf and R&D expenditure x, we can also determine non-R&D cost c by

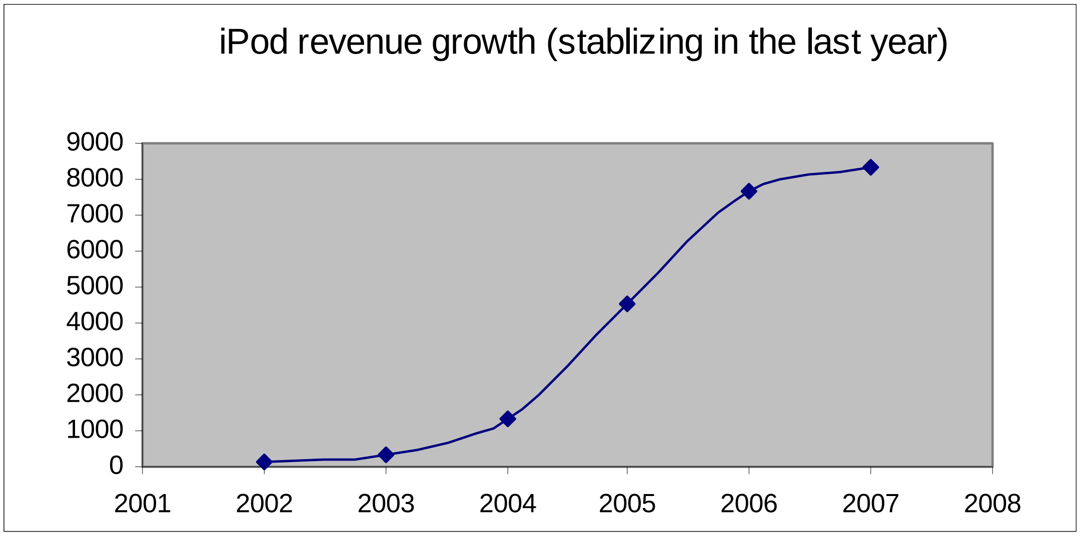
$$c = (1 - \text{operating margin})^{*}qf - x. \tag{EC.80}$$

MP3 Player Category: We use the monopoly model for this product because Apple iPod has market share in excess of 70% (Hall 2006). After introducing the first iPod in 2001, Apple has been launching new versions of the product and retiring old ones from the market every year (Apple-10K, 2001-2007), so we set $\tau^m = 1$ year.

Sales and revenues grew rapidly as the iPod penetrated the market in 2001-2005, but show signs of stabilizing in 2006 and 2007. As our model assumes stationarity, we use financial data collected since the product introduction in September 2006, which we believe is best represents the future, maturing market.

We are unable to find iPod-specific data on R&D expense and operating margin, and therefore we assume that R&D expense as a percentage of revenue and operating margin are 3% and 34%, respectively, corresponding to the overall figures for Apple (Apple 2007). Then (EC.75) and (EC.76) imply that

$$\alpha = 0.03 \text{ and } \beta = 0.36.$$



In fiscal year 2007, iPod unit sales is $N = 51.6$ million. The total revenue is \$8.305 billion, which implies R&D expenditure $x = \alpha q f = \$249$ million. From (EC.79) with $\tau = 1$ and (EC.80),

$$v = 4934 * 10^6, \text{ and } c = \$5.232 \text{ billion.}$$

Workstation Category:. The workstation category is also best represented by our monopoly model, because introduction of new workstations is driven by Intel's launching of new microprocessors in its Xeon line, and Intel has more than an 85% market share among producers of such microprocessors for workstations (Dignan 2007). Between Intel and workstation end-users are the workstation assemblers, such as Sun, Dell, and HP. The assembler's market is highly competitive and profits are slim. Intel is responsible for a majority of the R&D investment in the workstation product category, and captures a majority of the associated profit generated by new product introduction. Therefore, despite the complexity of the workstation supply chain, our simple monopoly model of Intel's new product introduction process is a reasonable approximation.

After introducing Pentium III Xeon in 1999, Intel launched Xeon MP in 2002 based on Pentium IV technology, and the Duo-Core and Quad-Core Xeons in March 2006. Thus we set $\tau^m = 3$ years.

We are not able to find public information on Intel's R&D expenditures for workstation (Xeon) microprocessors, and therefore take financial data for the company as a whole as representative. We use accumulated totals to average out small year-to-year variations. From 1999 to 2006, Intel generated \$254.98 billion in total revenue and spent \$34.99 billion on research and development (Intel-10K, 1999-2006), so from (EC.75)

$$\alpha = \frac{34.99}{254.94} = 0.137.$$

Fortunately, we did obtain more product-specific information about the operating margin. Intel's workstation microprocessor business is a part of the Digital Enterprise Group (DEG), for which revenue and operating income are broken out in Intel's annual reports (Intel-10K, 2003-2006). For the four financial years 2003-2006, DEG's total revenue is \$92.85 billion and total operating income is \$30.14 billion, which translates into an operating margin of 32.5%. Using (EC.76) with $\tau = 3$,

$$\beta = 0.387.$$

As Intel does not release specific revenue figures for its workstation (Xeon) microprocessor business, we estimate that revenue from scattered data on quarterly sales of workstations and microprocessor prices. Reports from a computer industry consulting firm, Jon Peddie Research, put the total number of workstations shipped in 2Q 2006 as $617,000$ units for 2Q 2006 (JPR 2006), $573,000$ units for 4Q 2006, and $674,000$ for 1Q 2007 (JPR 2007). Therefore we assume average quarterly sales of $600,000$ microprocessors, which translates, over the 3-year product lifetime, into sales of $N = 7.2$ million units per new product. Depending on processor variety, Xeon's retail price can range from \$200 to \$1000 or more (e.g. see price review and comparison at websites like www.dealtime.com). We use the median value of \$500 to set total revenue per new workstation processor $qf = 7.2 * 500 =$ \$3.6 billion. It follows that R&D expenditure is $x = \alpha qf =$ \$494 million. From (EC.79) with $\tau = 3$ and (EC.80)

$$v = 59.88 * 10^6 \text{ and } c = \$1.936 \text{ billion.}$$

Video Game Console Category: In the video game console market, Sony's Playstation battles Microsoft's Xbox. Committed gamers are the primary target consumers for each new generation of the Sony Playstation and Microsoft Xbox, whereas Nintendo has introduced a line of Wii consoles to appeal to a different demographic - casual or first-time gamers (Casey, 2006). Due to this separation in target consumer populations, we fit our duopoly model to represent the competition between Sony and Microsoft. The competition started in 2001 when Microsoft entered console market by introducing Xbox (Microsoft, 2001). Sony responded by introducing PlayStation2 Portable (PSP) in 2004 (SONY-20F, 2004). Microsoft introduced Xbox360 in 2005 (Microsoft, 2005), followed by Sony's introduction of PlayStation3 in 2006 (SONY-20F, 2006). Neither firm released a new product in 2007. Based on the average time between new product introductions, we set the duopoly equilibrium $\tau^d = 1.5$ years.

We obtain R&D expense and operating margin from a financial report for Sony's game business filed with the SEC. In Microsoft's annual report, the video console belongs to Home and Entertainment segment, within which the financial data is primarily determined by the software and is therefore not representative of the video game console business. From 2002 (when Microsoft entered the market) to 2007, Sony's game segment has incurred an R&D expense of 468.2 billion yen and generated a total revenue of 5444 billion yen with operating income of 82.8 billion yen (Sony-20F, 2002-2007), so R&D expense is 8.6% of the total revenue and the operating margin is 1.5%. The slim operating margin is consistent with reports on Microsoft that its video game consoles are unprofitable. Therefore we use the figures from Sony. From (EC.75) and (EC.78) with $\tau = 1.5$,

$$\alpha = 0.086 \text{ and } \beta = 0.915.$$

Our model assumes that users are homogeneous and therefore buy a new product at the time of its introduction. In reality, buyers of video game consoles are heterogeneous (an extension we address in §7) and therefore buy gradually over time as the manufacturer marks down prices. Thus we estimate total unit sales based on products that have already been retired, and do not use data for new products that are in the middle of the selling cycle. Specifically, we set total sales of a video game console at $N = 20$ million units corresponding to the total sales of Sony's PSP and Microsoft's Xbox. The price of a video game console can range from slightly below \$250 to \$600

(Kageyama, 2006), so we an approximate median price of \$400 to compute the revenue per new product $qf = \$400 * 20$ million = \$8 billion and then R&D expenditure $x = \alpha qf = \$0.48$ billion. By (EC.79) with $\tau = 1.5$ and (EC.80),

$$v = 698 * 10^6 \text{ and } c = \$7.192 \text{ billion}$$

Mobile Phone Category: We take a futuristic view of the U.S. mobile phone industry. Currently in the US, mobile phones are commonly bundled with a service agreement; service providers subsidize the phone purchase and recover revenue through subsequent service charges. However, the industry trend is to unbundle phone sales from service so that, in the near future, US consumers will buy mobile phones at market prices, independently of their choice of service provider (Segan, 2007; Holson, 2007; Searcey, 2007). We fit our duopoly model to this future "unbundled" scenario for the US market and determine parameter values by a different process from the above.

We assume in this future scenario, consumers will upgrade their mobile phones as frequently as they do today. The average lifespan of a mobile phone is 18 months (NY report 2004), which translates into $\tau = 1.5$ years.

We focus on US market of GSM phones where Nokia and Motorola are dominant. In the past three years, the two firms reported comparable R&D expense as percentage of total revenue ($\approx 10\%$) and operating margin ($10\% - 15\%$) for the entire company (income statements are at finance.yahoo.com). Nokia releases quarterly financial information that report data specific for mobile phones (Nokia, 2004-2007, no such detailed information is given in Motorola's reports), which we use to compute α and β. From 1Q 2004 to 3Q 2007, the total revenue of Nokia's mobile business is €87.746 billion, R&D expense is €4.673 billion, and operating income is €15.60 billion. This means R&D expenditure is 5.7% of total revenue and the operating margin is 18.4%.From (EC.75) and (EC.78) with $\tau = 1.5$,

$$\alpha = 0.057 \text{ and } \beta = 0.946.$$

GSM users in the US are primarily AT&T and T-Mobile subscribers. AT & T recently reported 65 million wireless subscribers (AT&T-10Q, 2007) and T-Mobile USA (T-Mobile-10Q, 2007) reports 28 million subscribers. We set $N = 94$ million, which is the combined total of the two companies. As of December 2007, unlocked mobile phones offered on Amazon website are divided into 5 price ranges, $\$50-\$99, \$100 - \$199, \$200 - \$499$, $\$500 - \$999$, and $\$1000 - \$1999$. The five categories have $10, 89, 201, 29, 9$ types of phones, respectively. Taking the middle point of each price range and calculate their weighted average (using the number of phone types as the weight), we set the average price of an unlocked phone at \$350, which coincide with the median of the most popular price range ($\$200 - \$499$). The product of the average price with the number of subscribers gives total revenue of $qf = \$32.9$ billion and R&D expense $x = \alpha qf = \$1.875$ billion. By (EC.79) and (EC.80), with $\tau = 1.5$,

$$v = 4836 \text{ and } c = \$24.97 \text{ billion.}$$

Table 1 Input Data and Fitted Model Parameter Values, for Four Categories of Electronic Products

	Monopoly		Duopoly	
	MP3 Player	Workstation	Video game console	Mobile phone (U.S. GSM)
Input data				
New product introduction interval τ (years)	1.0	3.0	1.5	1.5
R&D expense as % of revenue	3.0	13.7	8.6	5.7
Operating margin (%)	34.0	32.5	1.5	18.4
Total revenue (\$ billion)	8.3	3.6	8.0	32.9
Unit sales N (million)	51.6	7.2	20.0	94.0
Fitted model parameters				
Total non-R&D cost per new product c (\$billion)	5.232	1.936	7.192	24.97
Consumer utility per unit quality per year v (million)	4,934	59.88	698.7	4,836
Quality sensitivity to R&D expenditure α	0.030	0.137	0.086	0.057
Quality sensitivity to development time β	0.361	0.387	0.915	0.946

Supplement to §7

Due to page limitations, §7 provided only a short description of our model of heterogeneous consumers, and stated our results briefly and informally. This supplement gives more detail about our model of heterogeneous consumers, shows how to compute the fractional-revenue parameter θ, and then provides proofs for all the claims made in §7.

Modeling Consumer Heterogeneity

Following (Gul et al 1986), for our unit mass of consumers, let $u:[0,1]\rightarrow R_+$ denote utility per unit quality improvement q per unit time, where u is non-increasing, left-continuous, Lipschitz at 1, and satisfies $\int_0^1 u(y)dy = 1$ and $u(1)qf > c$, meaning that all consumers value the new product at more than its production cost. Suppose that the firm is able to mark down its price sufficiently rapidly to sell to all consumers before the next new product is introduced. Given that the firm will sell the new product to all consumers, the total amount produced is 1. Suppose that this production occurs in a single lot at the time of new product introduction, so that the discounted production cost is independent of the price reduction process. Let Δ be the minimum time between changes in price, specified exogenously as in (Gul et al, 1986), so the firm is able to change its price at times $0, \Delta, 2\Delta, 3\Delta, ...$ When the firm announces a new price, each consumer decides whether to pay that price and obtain the product instantaneously, or wait to purchase in hope of paying a lower price. We focus on open-loop stationary equilibria, in which the price process and consumer purchasing behavior is identical in each new product introduction cycle. Specifically, each consumer $y \in [0,1]$ will purchase at the same point in time in each new product introduction cycle, and therefore use each new product for precisely the equilibrium new product introduction time and have the same qf as all other consumers. Therefore variation in consumers' willingness to pay for a new product is captured entirely by the utility multiplier $u(y)$.

First consider the case that $qf = 1$, so that consumer $y's$ utility is $u(y)$. Theorem 1 and its corollary in Section 3 of (Gul et al 1986) establish that there exists a unique equilibrium in which the firm sets price P_j and customers in segments $[y_j, y_{j+1})$ buy the new product at time $j\Delta$, for $j = 0, 1, 2, ..., N-1$, where N is finite and $y_0 = 0$. (Our assumption that the firm is able to mark down its price sufficiently rapidly to sell to all consumers before the next new product is introduced is, in this equilibrium notation, that $(N-1)\Delta$ is less than the equilibrium time between new product introductions.) From Lemmas 4 and 5, and the proof of Theorem 1 of (Gul et al, pp. 176-184), this equilibrium is characterized by the firm's optimality condition:

$$\max_{N,\ \{y_j\}_{j=1..,N-1}} \left\{ \sum_{j=0}^{N-1} P_j (y_{j+1} - y_j) \rho^j \right\} \tag{EC.81}$$

$$\text{subject to: } P_j = (1-\rho) u(y_{j+1}) + \rho P_{j+1},\ j = 0, 1, ..., N-2, \tag{EC.82}$$

where $\rho \equiv \delta \Delta$. (As in the paper, δ denotes the discount rate per unit of time, so over a markdown period of length Δ, consumer utility and the firm's revenues are discounted by δ.) The objective function in (EC.81) is the firm's discounted revenue on each new product, and the constraint (EC.82) ensures that the consumer with lowest utility who buys at price P_j is just willing to buy at that price, rather than wait for time Δ to buy at price P_{j+1}. In the equilibrium, prices are strictly decreasing and all consumers are served, i.e., $P_0 > P_1 > ... > P_{N-1} = u(1)$ and $y_N = 1$.

Observe that (EC.81) and (EC.82) constitute a homogeneous system, so that if $[(y_1, ...\ y_N), (P_0, ..P_{N-1})]$ is the equilibrium for utility function $u(y)$, then

$$[(y_1, ...y_N), (qfP_0, qfP_1, ..qfP_{N-1})]$$

is the equilibrium when the utility function is $u(y)qf$. Hence for any qf and c, the firm's discounted revenue is

$$qf \sum_{j=0}^{N-1} P_j (y_{j+1} - y_j) \rho^j = \theta qf \tag{EC.83}$$

where $[(y_1, ...\ y_N), (P_0, ..P_{N-1})]$ are obtained by solving (EC.81)-(EC.82) and

$$\theta \equiv \sum_{j=0}^{N-1} P_j (y_{j+1} - y_j) \rho^j$$

is therefore a positive constant that is independent of qf and c. Because $P_{N-1} = u(1)$ and because of (EC.82)

$$P_j (y_{j+1} - y_j) \leq u(y_{j+1})(y_{j+1} - y_j).$$

Hence

$$\theta = \sum_{j=0}^{N-1} P_j(y_{j+1} - y_j)\rho^j \le \sum_{j=0}^{N-1} u(y_{j+1})(y_{j+1} - y_j) \le \int_0^1 u(y)dy = 1$$

and $\theta = 1$ is reached when customers are homogeneous, i.e., $u(y) = 1$ for all $u \in [0,1]$ so that $N = 1$. Theorem 3 in section 5 of (Gul et al 1986) implies that as Δ converges to zero, the initial equilibrium price converges to $u(1)qf$ and therefore θ converges to $u(1)$, which is the Coase conjecture.

We now show how consumer heterogeneity affects the equilibrium profit, consumer surplus and social welfare. Let $\tau (= \tau^m$ in the monopoly model and $= \tau^d$ in the duopoly model) denote the equilibrium new product introduction time. Expression (EC.83) for the equilibrium discounted revenue on each new product implies that total discounted profit on all new product introductions (in the duopoly case, the profit of both firms) is

$$\frac{e^{-\delta\tau}}{1 - e^{-\delta\tau}}(\theta qf - x - c); \tag{EC.84}$$

note that the expressions for monopoly and duopoly discounted profit in our basic model with homogeneous consumers, π^m and π^d in the proof of Proposition 7, are special cases of (EC.84) with $\theta = 1$. A consumer in segment $[y_j, y_{j+1})$ $(0 = y_0 < y_1 < ... < y_N = 1)$ pays qfP_j for a new product and obtains utility $qu(y)$ per unit of time. As the incremental quality q associated with the current product is carried over into all subsequent new products, and in equilibrium the consumer purchases every new product, the consumer's discounted utility associated with the current new product is

$$\int_0^{+\infty} qu(y)e^{-\delta\tau}d\tau = \frac{q}{\delta}u(y)$$

and total consumer surplus associated with the current new product, discounted to the time at which the current new product is introduced, is

$$\sum_{j=0}^{N-1} \int_{y_j}^{y_{j+1}} \left(\frac{q}{\delta}u(y) - qfP_j\right)\rho^j dy = \lambda\frac{q}{\delta} - \theta qf, \text{ where } \lambda = \sum_{j=0}^{N-1} \int_{y_j}^{y_{j+1}} u(y)\rho^j dy.$$

Because $u(y) \ge P_j$ $(y \in [y_j, y_{j+1}),\ j = 0, 1, ..., N-1)$ in equilibrium, the fractional-revenue parameter θ and the consumer surplus parameter λ satisfy

$$\theta \le \lambda.$$

Consumer surplus associated with all new product introductions is

$$CS = \left(\lambda\frac{q}{\delta} - \theta q f\right)\left(e^{-\delta\tau} + e^{-2\delta\tau} + ...\right) = \frac{e^{-\delta\tau}}{1-e^{-\delta\tau}} q\left(\frac{\lambda}{\delta} - \theta f\right). \quad \text{(EC.85)}$$

In the special case of homogeneous consumers ($u(y) = 1$ for all $y \in [0,1]$), $\lambda = \theta = 1$ and (EC.85) reduces to the expressions for consumer surplus for the duopoly model (EC.52) and monopoly model (EC.53) in the proof of Proposition 9. In equilibrium, consumers dispose of a product τ units of time after purchasing it, so the environmental, health and processing costs of e-waste associated with a new product, discounted to the time at which that new product is introduced, are

$$\sum_{j=0}^{N-1}\int_{y_j}^{y_{j+1}}(k+z)e^{-\delta\tau}\rho^j dy = \psi(k+z)e^{-\delta\tau}, \text{ where } \psi = \sum_{j=0}^{N-1}(y_{j+1}-y_j)\rho^j.$$

The social welfare generated by each new product is the sum of profit and consumer surplus, less the environmental, health and processing costs for e-waste

$$\lambda\frac{q}{\delta} - x - c - \psi(k+z)e^{-\delta\tau},$$

and social welfare from all new product introductions is

$$\begin{aligned} W &= \left(\lambda\frac{q}{\delta} - x - c - \psi(k+z)e^{-\delta\tau}\right)\left(e^{-\delta\tau} + e^{-2\delta\tau} + ...\right) \\ &= \frac{e^{-\delta\tau}}{1-e^{-\delta\tau}}\left[\frac{\lambda}{\delta}q((1+\gamma)\tau, x) - x - c - \psi e^{-\delta\tau}(k+z)\right]. \end{aligned} \quad \text{(EC.86)}$$

Propositions and Proofs

We now restate the informal claims made in §7 and prove those claims. In both the fractional-revenue case with heterogeneous consumers and the fractional-profit case with multiple firms in the supply chain, let θ be the fraction. The following statements apply to both the fractional-revenue and the fractional-profit cases

1. "*All propositions in §2 and §3 remain true for general* $\theta \in (0,1]$."

In the fractional-revenue case, the proofs of Propositions 1-5 (for the case $\theta = 1$) hold for general case $\theta \in (0,1]$, with the simple substitution of $\theta q(\tau, x)$ for the incremental quality function $q(\tau, x)$. Note that our assumptions about q in Equations 1-5 and Equation 16 imply that the same assumptions hold for θq. Our assumption that the business is potentially profitable for the monopolist requires that (8) holds with the substitution of $\theta q(\tau, x)$ for $q(\tau, x)$.

Similarly, in the fractional-profit case, the proofs of Propositions 1-5 (for the case $\theta = 1$) hold for general case $\theta \in (0, 1]$, with the simple substitution of $\theta q(\tau, x)$ for the incremental quality function $q(\tau, x)$ and θc for the production cost c.

2. "*In both the monopoly model and the duopoly model, the unique equilibrium time between new product introductions decreases with* θ*, which means that customer heterogeneity and multiple firms in the supply chain cause a slow-down in new product introduction.*"

This claim is stated formally in Proposition 12 for the monopoly and Proposition 13 for the duopoly, and proofs are provided for those propositions, below.

3. "*All propositions in §4 hold as stated for general* $\theta \in (0, 1]$*, with one exception: a fee-upon-sale is no longer guaranteed to increase social welfare in the duopoly model, as stated in the first part of Proposition 8. A fee-upon-sale increases social welfare when* θ *is large, but may decrease social welfare when* θ *is small and the production cost* c *is large.*"

In the fractional-revenue case, the proof of Propositions 6 (for the case $\theta = 1$) holds for the general case $\theta \in (0, 1]$, with the simple substitution of $\theta q(\tau, x)$ for the incremental quality function $q(\tau, x)$. In the fractional-profit case, the proof of Propositions 6 (for the case $\theta = 1$) hold for general case $\theta \in (0, 1]$, with the simple substitution of $\theta q(\tau, x)$ for the incremental quality function $q(\tau, x)$ and θc for the production cost c. We know from Proposition 8 that a fee-upon-sale strictly increases social welfare in the duopoly model when $\theta = 1$. The claim that the fee-upon-sale may decrease social welfare in the duopoly model when θ is small and the production cost c is large is stated formally in Proposition 14 and then proven, below. The proof of Proposition 8 for the monopoly model holds for general $\theta \in (0, 1]$ with the substitution of (EC.86) for social welfare in the fractional-revenue (heterogeneous consumers) case, and with the expression for social welfare unchanged in the fractional-profit case. Propositions 9 and 7 are addressed in points 4 and 5 immediately following.

4. "*Even though heterogeneity enables consumers to capture more of that value, Proposition 9 holds: a fee-upon-sale decreases consumer surplus.*"

Imposing a fee-upon-sale is equivalent to increasing c, so this claim follows from Proposition 15, which is stated and proven below.

5. "*In Proposition 7 the thresholds* c^m_{profit} *and* c^d_{profit} *increase with* θ."

Proposition 7 holds as stated; the additional information provided here is that the threshold parameters increase with θ, which is stated formally in Proposition 16 and then proven, below. The result in Proposition 7 that a fee-upon-disposal strictly decreases the duopolist's profit follows immediately from Proposition 6 for general $\theta \in (0,1]$, that the duopoly equilibrium timing and expenditure in new product introduction are unchanged. .

6. "*Proposition 10 in §6 holds for general* $\theta \in (0,1]$. *However, in the duopoly model with fractional profit (multiple firms in the supply chain) and* $\theta < 1$, *individual EPR fails to give socially optimal incentives for investment to reduce the end-of-life processing cost.*"

In the fractional-revenue case, the proof of Propositions 10 (for the case $\theta = 1$) holds for the general case $\theta \in (0,1]$, with the simple substitution of $\theta q(\tau, x)$ for the incremental quality function $q(\tau, x)$. In the fractional-profit case, the proof of Propositions 10 (for the case $\theta = 1$) hold for general case $\theta \in (0,1]$, with the simple substitution of $\theta q(\tau, x)$ for the incremental quality function $q(\tau, x)$, θc for the production cost c, and θk for end-of-life cost k, except where k is the argument of $I()$, as the innovating firm bears the full investment $I()$ in design for recyclability. For $\theta = 1$ under individual EPR, Proposition 10 implies that the duopolists make the socially optimal investment to reduce the end-of-life cost. However, for $\theta < 1$ in the fractional-profit case under individual EPR, the duopolist will invest strictly less to reduce the end-of-life cost. The end-of-life cost that minimizes a duopolist's total discounted cost will be strictly greater than the socially optimal level, because the duopolist bears the entire cost of investment to reduce that end-of-life cost $I(k)$, but its supply chain partners bear the fraction $(1-\theta)$ of the end-of-life cost k.

7. "*..the monopolist's investment is not necessarily monotone in* θ."

The is a direct corollary from Proposition 17 (which shows there exist cases in which the monopolist makes no investment when θ is large but some investment when θ is small) and Proposition 18 (which shows there also exist cases in which the monopolist's investment increases with θ). Propositions 17 and 18 are stated and proven below.

Proposition EC.2 *In both the fractional-profit and fractional-revenue cases, let θ be the value of the fraction. In the monopoly model with Cobb-Douglas quality function, if τ_i^m is the equilibrium development time reached under θ_i $(i=1,2)$ and $\theta_1<\theta_2$, then $\tau_1^m>\tau_2^m$.*

Proof: We first consider the fractional-profit case where the monopoly profit from a new product is $\theta(qf-c)-x$, and the first-order conditions for equilibrium are

$$\theta q_\tau(\tau,x)f(\tau)-\theta q(\tau,x)+\frac{x+\theta c}{f}=0 \tag{EC.87}$$

$$\theta q_x(\tau,x)f(\tau)=1. \tag{EC.88}$$

We prove the conclusion by contradiction. If $\theta_1<\theta_2$ and $\tau_1^m\le\tau_2^m$, then to satisfy (EC.88) for both θ_1 and θ_2, $q_x(\tau_1^m,x_1^m)>q_x(\tau_2^m,x_2^m)$, and thus $x_1^m<x_2^m$ and $q_1^m<q_2^m$.

In (EC.87), replace x by $\alpha q/q_x(=\alpha q\theta f)$ and q_τ by $\beta q/\tau$,

$$\theta q_\tau f-\theta q+\frac{x+\theta c}{f}=\frac{\theta}{f}\left[qf\left(\beta\frac{f}{\tau}-1+\alpha\right)+c\right]=0.$$

For the above to hold for both θ_1 and θ_2,

$$c=q_1^m f(\tau_1^m)\left(1-\beta\frac{f(\tau_1^m)}{\tau_1^m}-\alpha\right)=q_2^m f(\tau_2^m)\left(1-\beta\frac{f(\tau_2^m)}{\tau_2^m}-\alpha\right) \tag{EC.89}$$

which cannot be true if $\tau_1^m\le\tau_2^m$ and $q_1^m<q_2^m$ because $f(\tau)$ and $-f(\tau)/\tau$ both increase in τ.

In the fractional-revenue case, the monopoly profit from a new product is $\theta(qf-c)-x$ to $\theta qf-c-x$. The first-order condition with respect to τ is

$$\theta q_\tau(\tau,x)f(\tau)-\theta q(\tau,x)+\frac{x+c}{f}=0$$

and the first-order condition with respect to x is the same as (EC.88). Using these two first-order conditions and following the same steps that lead to (EC.89),

$$c=\theta_1 q_1^m f(\tau_1^m)\left(1-\beta\frac{f(\tau_1^m)}{\tau_1^m}-\alpha\right)=\theta_2 q_2^m f(\tau_2^m)\left(1-\beta\frac{f(\tau_2^m)}{\tau_2^m}-\alpha\right)$$

which also cannot be true if $\theta_1<\theta_2$ and $\tau_1^m\le\tau_2^m$. ■

Proposition EC.3 *In both the fractional-profit and fractional-revenue cases, let θ be the value of the fraction. In the duopoly model with Cobb-Douglas quality function, if τ_i^d is the equilibrium time between new product introductions asociated with θ_i $(i = 1, 2)$ and $\theta_1 < \theta_2$, then $\tau_1^d > \tau_2^d$.*

Proof: In the fractional-profit case, the first-order conditions for duopoly equilibrium are

$$\theta q_\tau f - \theta q + \delta(x + \theta c) = 0 \text{ and } \theta q_x f = 1.$$

By the use of the latter condition with $q_x = \alpha q/x = a x^{\alpha-1}\tau^\beta$,

$$x = \left\{\alpha\theta f[(1+\gamma)\tau]^\beta\right\}^{1/(1-\alpha)} \text{ and } q = (\alpha\theta f)^{\alpha/(1-\alpha)}\,[(1+\gamma)\tau]^{\beta/(1-\alpha)}$$

By transforming the first-order condition for τ,

$$\begin{aligned}\delta c &= q - q_\tau f - \delta\frac{x}{\theta}\\ &= (\alpha\theta)^{\alpha/(1-\alpha)} f^{\alpha/(1-\alpha)}[(1+\gamma)\tau]^{\beta/(1-\alpha)}\left(1 - \beta\frac{f(\tau)}{(1+\gamma)\tau} - \alpha\delta f(\tau)\right)\\ &= \theta^{\alpha/(1-\alpha)}\widetilde{H}_d(\tau),\end{aligned} \tag{EC.90}$$

where $\widetilde{H}_d(\tau) = q - q_\tau f - \delta x(\tau)$ where

$$\begin{aligned} x(\tau) &= (\alpha f(\tau))^{1/(1-\alpha)}[(1+\gamma)\tau]^{\beta/(1-\alpha)},\\ q &= x^\alpha[1+\gamma)\tau]^\beta.\end{aligned}$$

Observe that $q_x f = 1$ at $x = x(\tau)$. Now we compare $\widetilde{H}_d(\tau)$ with $H_d(\tau)$ in the proof of Proposition 4 in the paper (page 41, Equation 61),

$$\widetilde{H}_d(\tau) = -H_d(\tau) + \delta c.$$

Hence from (EC.90),

$$-H_d(\tau) = (\theta^{-\alpha/(1-\alpha)} - 1)\delta c. \tag{EC.91}$$

We show in the proof of Proposition 4 that $H_d(\tau)$ strictly decreases in τ, so $-H_d(\tau)$ strictly increases in τ. Because the right-hand side decreases in θ, this means if $\theta_1 < \theta_2$, then $\tau_1^d > \tau_2^d$.

To prove the fractional-revenue case, we follow the same steps that lead to (EC.91) to establish that when the profit from a new product is $\theta q f - x - c$,

$$-H_d(\tau) = (\theta^{-1/(1-\alpha)} - 1)\delta c.$$

Therefore $\tau_1^d > \tau_2^d$ if $\theta_1 < \theta_2$. ■

Proposition EC.4 *In the duopoly model with Cobb-Douglas quality function, if c is sufficiently large, then imposing a small fee-upon-sale can strictly decrease social welfare in the presence of heterogeneous customers or multiple firms in the supply chain.*

Proof: As in our formulation above, social welfare in the duopoly model is

$$W^d = \frac{e^{-\delta\tau^d}}{1-e^{-\delta\tau^d}}\left[\frac{\lambda}{\delta}q((1+\gamma)\tau^d, x^d) - x^d - c - \psi e^{-\delta\tau^d}(k+z)\right]$$

where in the presence of heterogeneous customers, $0 < \theta < \lambda \leq 1$. A fee-upon-sale is a transfer from manufacturers to other sectors of economy, hence does not have a *direct* impact on social welfare. Nevertheless, imposing a fee will increases equilibrium development time, so social welfare strictly decreases if $dW^d/d\tau^d < 0$.

$$\begin{aligned}\frac{dW^d}{d\tau^d} &= \frac{e^{-\delta\tau^d}}{(1-e^{-\delta\tau^d})^2}[-\lambda q + \delta(x^d + c) + (1+\gamma)\lambda q_\tau f \\ &\quad + \delta\psi e^{-\delta\tau^d}(2-e^{-\delta\tau^d})(k+z) + \left(\frac{\lambda}{\delta}q_x - 1\right)\delta f\frac{dx^d}{d\tau^d}]\end{aligned} \tag{EC.92}$$

Apply Implicit Function Theorem to the first-order condition $\theta q_x = 1$,

$$\frac{dx^d}{d\tau^d} = \frac{(1+\gamma)q_{\tau x}f + q_x f_\tau}{-q_{xx}f} = \frac{\alpha(1+\gamma)q_\tau f + \alpha q f_\tau}{(1-\alpha)q_x f} = \frac{\alpha\theta}{1-\alpha}((1+\gamma)q_\tau f + q f_\tau). \tag{EC.93}$$

Insert the above in to the term in the square bracket of (EC.92),

$$\begin{aligned}&-\lambda q + \delta(x^d + c) + (1+\gamma)\lambda q_\tau f + \left(\frac{\lambda q_x}{\delta} - 1\right)\delta f\frac{dx^d}{d\tau^d} + \delta\psi e^{-\delta\tau^d}(2-e^{-\delta\tau^d})(k+z) \\ = &-\lambda q + \delta(x^d + c) + (1+\gamma)\lambda q_\tau f + \frac{\alpha}{1-\alpha}(\lambda - \theta\delta f)((1+\gamma)q_\tau f + q f_\tau) \\ &\quad + \delta\psi e^{-\delta\tau^d}(2-e^{-\delta\tau^d})(k+z)\end{aligned} \tag{EC.94}$$

and imposing a fee-upon-sale strictly decreases social welfare if (EC.94) is strictly negative. Use the first-order condition of τ

$$-q+q_\tau f+\delta(x/\theta+c)=0$$

to eliminate c, and substitute x^d with $\alpha\frac{q}{q_x}=\alpha\theta q f$ and q_τ with $\beta\frac{q}{(1+\gamma)\tau^d}$, the right-hand side of (EC.94) becomes

$$\begin{aligned}
&q(1-\lambda)-(1/\theta-1)\,\delta x^d-[1-(1+\gamma)\lambda]q_\tau f+\frac{\alpha(\lambda-\theta\delta f)}{1-\alpha}\left((1+\gamma)q_\tau f+f_\tau\right)\\
&+\delta\psi e^{-\delta\tau^d}(2-e^{-\delta\tau^d})(k+z)\\
=\;& q\left\{(1-\lambda)-\alpha(1-\theta)\delta f-\left[\frac{1}{(1+\gamma)}-\lambda\right]\beta\frac{f}{\tau^d}+\frac{\alpha(\lambda-\theta\delta f)}{1-\alpha}\left(\beta\frac{f}{\tau^d}+f_\tau\right)\right\}\\
&+\delta\psi e^{-\delta\tau^d}(2-e^{-\delta\tau^d})(k+z).
\end{aligned}$$

When $\tau^d\to+\infty$, $f_\tau(=e^{-\delta\tau^d})\to 0$, $f/\tau^d\to 0$, and $\delta f\to 1$. Hence if $\theta<1$ and $\alpha>(1-\lambda)/(1-\theta)$, then $dW^d/d\tau^d<0$ when τ^d becomes sufficiently large.

In case of multiple firms in supply chain, $\lambda=\psi=1$ and (EC.92) becomes

$$\begin{aligned}
\frac{dW^d}{d\tau^d}=\;&\frac{e^{-\delta\tau^d}}{(1-e^{-\delta\tau^d})^2}[-q+\delta(x^d+c)+(1+\gamma)q_\tau f\\
&+\delta e^{-\delta\tau^d}(2-e^{-\delta\tau^d})(k+z)+\left(\frac{1}{\delta}q_x-1\right)\delta f\frac{dx^d}{d\tau^d}].
\end{aligned}$$

use (EC.93) to remove $dx^d/d\tau^d$, the first-order condition

$$-\theta(q-q_\tau f)+\delta(x+c)=0$$

to remove $\delta(x^d+c)$, and substitute q_τ with $\beta\frac{q}{(1+\gamma)\tau^d}$, $dW^d/d\tau^d$ strictly decreases if

$$q\left[-(1-\theta)+\frac{(1-\theta+\gamma)f}{(1+\gamma)\tau^d}+\frac{\alpha(1-\theta\delta f)}{1-\alpha}\left(\beta\frac{f}{\tau^d}+f_\tau\right)\right]+\delta e^{-\delta\tau^d}(2-e^{-\delta\tau^d})(k+z)<0$$

which is true if $\theta<1$ and τ^d is sufficiently large.

In both cases, by the first-order condition of τ, a sufficiently large θ can be obtained as a result of a sufficiently large c. ■

Proposition EC.5 *In the duopoly model with Cobb-Douglas quality function and heterogeneous customers, consumer surplus strictly decreases with c. In the monopoly model with Cobb-Douglas quality function and heterogeneous customers, consumer surplus strictly decreases with c if $c > c^m_{cs}$. The condition $c \le c^m_{cs}$ implies that $c < x^m$.*

Proof: From the problem formulation, consumer surplus in the heterogeneous customer case is

$$CS = \frac{e^{-\delta\tau}}{1-e^{-\delta\tau}} q \left(\frac{\lambda}{\delta} - \theta f\right).$$

where $\tau = \tau^m$ in the monopoly model and $\tau = \tau^d$ in the duopoly model.

A fee-upon-sale does not impact consumer surplus directly, but by Proposition 6, will extend equilibrium new product introduction time. Hence consumer surplus strictly decreases if

$$\frac{dCS}{d\tau} < 0.$$

Take the derivative of CS and rearrange terms,

$$\begin{aligned} \frac{dCS}{d\tau} &= \frac{-e^{-\delta\tau}}{(1-e^{-\delta\tau})^2} q\,(\lambda - \theta\delta f) + \frac{e^{-\delta\tau}}{\delta(1-e^{-\delta\tau})}\left[\left(\frac{\partial q}{\partial \tau} + q_x \frac{dx}{d\tau}\right)(\lambda - \theta\delta f) - \theta q \delta f_\tau\right] \\ &= \frac{e^{-\delta\tau}(\lambda - \theta\delta f)}{(1-e^{-\delta\tau^d})^2}\left[-q\left(1 + \frac{\theta\delta f_\tau}{(\lambda - \theta\delta f)} f\right) + \left(\frac{\partial q}{\partial \tau} + q_x \frac{dx}{d\tau}\right) f\right] \end{aligned} \tag{EC.95}$$

In the duopoly case, apply (EC.93) to (EC.95) and use $(1+\gamma)q_\tau = \partial q/\partial\tau = \beta q/\tau^d$, (EC.95) is negative if and only if

$$\left(\frac{1}{1-\alpha}\right)\frac{\beta}{\tau^d} f + \frac{\alpha}{1-\alpha} f_\tau < \left(1 + \frac{\theta\delta f_\tau}{(\lambda - \theta\delta f)} f\right)$$

which is true because from the first-order condition on τ^d

$$\theta q_\tau f - \theta q + \delta(x + c) = 0$$

and the break-even condition: $\theta q f - x - c \ge 0$,

$$q_\tau f \ge q f_\tau, \text{ i.e., } \beta \frac{f}{(1+\gamma)\tau^d} \ge f_\tau. \tag{EC.96}$$

Hence

$$\left(\frac{1}{1-\alpha}\right)\frac{\beta}{\tau^d} f + \frac{\alpha}{1-\alpha} f_\tau \le \frac{f}{\tau^d} + \left(\frac{1}{\beta} - 1\right) f_\tau \quad \le 2\frac{f}{\tau^d} - f_\tau < 1$$

where the first inequality is due to $1-\alpha \geq \beta$ and the second inequality is due to (EC.96). To verify the last inequality, let $y=\delta\tau^d$

$$2\frac{f}{\tau^d} - f_\tau = \frac{2(1-e^{-y})}{y} - e^{-y}$$

which is less than 1 for all $y \geq 0$.

In the monopoly case, use (EC.88) and apply the Implicit Function Theorem, under Cobb-Douglas formulation

$$q_x \frac{dx^m}{d\tau^m} f = \frac{1}{\theta}\frac{dx^m}{d\tau^m} = \frac{q_{\tau x} f + q_x f_\tau}{-q_{xx} f\theta} = \frac{\alpha(q_\tau/x)f + \alpha(q/x)f_\tau}{(1-\alpha)(q_x/x)f} = \frac{\alpha}{1-\alpha}(q_\tau f + q f_\tau)$$

Use the above to substitute $q_x \frac{dx}{d\tau} f$ and $\beta q/\tau$ to substitute $\partial q/\partial\tau$ in (EC.95), $dCS/d\tau$ is strictly negative at τ^m if

$$\frac{\beta}{1-\alpha}\frac{f}{\tau^m} + \frac{\alpha f_\tau}{1-\alpha} \leq 1. \tag{EC.97}$$

Because the left-hand side of the inequality strictly decreases in τ^m and τ^m increases in c, there exists some threshold c^m_{cs} such that the inequality is satisfied if and only if $c > c^m_{cs}$. To show this is always the case when $x^m < c$,apply Cobb-Douglas formulation and substitute q_x with $\alpha q/x^m$ and q_τ with $\beta q/\tau^m$ in the first-order conditions

$$\theta q_\tau f - \theta q + \frac{x^m + c}{f} = 0 \text{ and } \theta q_x f = 1,$$

we arrive at

$$\beta\frac{f}{\tau^m} = 1 - \frac{x^m}{\theta q f} - \frac{c}{\theta q f} = 1 - \frac{x^m q_x}{q} - \frac{c}{\theta q f} = 1 - \alpha - \frac{c}{\theta q f}.$$

Insert the above into (EC.97),

$$\frac{\beta}{1-\alpha}\frac{f}{\tau^m} + \frac{\alpha f_\tau}{1-\alpha} = 1 - \frac{c}{(1-\alpha)\theta q f} + \frac{\alpha f_\tau}{1-\alpha} = 1 - \frac{c - x^m f_\tau}{(1-\alpha)\theta q f} < 1. \quad \blacksquare$$

Proposition EC.6 *In both the fractional-profit and fractional-revenue cases, let θ be the value of the fraction. In either the monopoly or duopoly model with Cobb-Douglas quality function, if imposing a fee-upon-sale does not improve profit when $\theta = \theta_2$, it will not improve profit for any $\theta_1 < \theta_2$.*

Proof: For the monopoly case, the proof of Proposition 7 of the paper up to Equation EC.43 (page 45-46) applies to both fractional-profit and fractional-revenue cases where the monopoly profit on a new product is $\theta(qf-c)-x$ and $\theta qf-x-c$ respectively. Parameter θ cancels out during the algebraic transformation. Hence, as in Equation EC.43 , imposing a small fee-upon-sale improves the profit if and only if

$$1-\frac{2-\alpha}{1-\alpha}f_\tau+\frac{f}{\tau^m}\left(1-\frac{\beta}{1-\alpha}\right)<0.$$

Since the left-hand side increases in τ^m and by Proposition 12, $\tau_1^m>\tau_2^m$ when $\theta_1<\theta_2$, the above must be true when $\theta=\theta_2$ if it holds for $\theta=\theta_1$.

Similarly, for the duopoly case, the proof of Proposition 7 up to Equation EC.46 (page 49) applies. So there is a unique upper bound (that is independent of θ) on τ^d. Duopoly profit improves if and only if τ^d is below that bound. Because τ^d decreases in θ, if the profit improves when $\theta=\theta_1$, it must improve when $\theta=\theta_2$. ■

Proposition EC.7 *In both the fractional-profit and fractional-revenue cases, let θ be the value of the fraction. In the monopoly model with Cobb-Douglas quality function, if investing a non-zero amount in recyclability increases the firm's profit when $\theta=\theta_2$, then the same is true for all $\theta_1<\theta_2$.*

Proof: In the fractional-profit case, the monopolist chooses $k\in(-c,\bar{k})$ to maximize

$$-I(k)+L(\tau^m,x^m)-\frac{e^{-\delta\tau^m}}{1-e^{-\delta\tau^m}}\theta k$$

$$\text{where}\quad L(\tau^m,x^m)=\frac{e^{-\delta\tau^m}}{1-e^{-\delta\tau^m}}[\theta q(\tau^m,x^m)f(\tau^m)-x^m-\theta c] \tag{EC.98}$$

the monopolist has no incentive to invest if and only if

$$\frac{dL}{d\tau^m}\frac{d\tau^m}{dk}-\frac{\theta e^{-\delta\tau^m}}{1-e^{-\delta\tau^m}}>0.$$

From the proof of Proposition 7 in the paper (page 46, Equation EC.43), the above is true if and only if $\tau^m<\bar{\tau}^m$ where $\bar{\tau}^m$ is the unique solution of

$$\left(1-\frac{\beta}{1-\alpha}\right)=\frac{\bar{\tau}^m}{f}\left(\frac{2-\alpha}{1-\alpha}f_\tau-1\right).$$

Since τ^m decreases in θ, $\tau^m < \bar{\tau}^m$ for θ_1 implies $\tau^m < \bar{\tau}^m$ for θ_2.

Similarly, in the fractional-revenue case, the monopolist chooses $k \in (-c, \bar{k})$ to maximize

$$-I(k) + L(\tau^m, x^m) - \frac{e^{-\delta\tau^m}}{1 - e^{-\delta\tau^m}} k$$

where with slight abuse of notation, we replace θc with c in (EC.98) for the definition of L. The monopolist has no incentive to invest if $\tau^m < \bar{\tau}^m$. The result follows because $\bar{\tau}^m$ is independent of θ and τ^m decreases in θ. ■

Proposition EC.8 *In both the fractional-profit and fractional-revenue cases, let θ be the value of the fraction. If $c + k$ is sufficiently large, a monopolist will always invest in recyclability, the investment in either case is smaller than k^*, the amount that minimizes EOL cost. Moreover, for sufficiently large c, under standard Cobb-Douglas function, the amount of investment increases in θ.*

Proof: When $c > c^m_{profit}$, $\bar{\tau}^m < \tau^m$ for all $\theta \leq 1$, so a monopolist always invests in both fractional-profit and fractional-revenue cases. In the fractional-profit case, the optimal amount of investment is determined by

$$-I'(k) = \frac{e^{-\delta\tau^m}}{1 - e^{-\delta\tau^m}}\theta - \frac{dL}{d\tau^m}\frac{d\tau^m}{dk}. \tag{EC.99}$$

where L is defined in (EC.98). Following the same steps in the proof of Proposition 7 (from Equation 64 to Equation 66, page 45-46), for the fractional-profit case, the right-hand side of (EC.99) becomes

$$\begin{aligned}
\frac{e^{-\delta\tau^m}}{1 - e^{-\delta\tau^m}}\theta - \frac{dL}{d\tau^m}\frac{d\tau^m}{dk} &= \frac{e^{-\delta\tau^m}}{1 - e^{-\delta\tau^m}}\left[1 - \frac{qf_\tau}{qf_\tau + q_\tau f\left(1 - \frac{2-\alpha}{1-\alpha}f_\tau\right)}\right]\theta \\
&= \frac{\theta}{\delta}\frac{q_\tau\left(1 - \frac{2-\alpha}{1-\alpha}f_\tau\right)}{q + q_\tau f\left(e^{\delta\tau^m} - \frac{2-\alpha}{1-\alpha}\right)} \\
&= \frac{\theta}{\delta}\frac{[1 - \alpha - (2-\alpha)f_\tau]}{\tau^m + (e^{\delta\tau^m} - 1)[1 - \alpha - (2-\alpha)f_\tau]} \\
&= \frac{\theta}{\delta}\left(\frac{\tau^m}{1 - \alpha - (2-\alpha)f_\tau} + e^{\delta\tau^m} - 1\right)^{-1}.
\end{aligned} \tag{EC.100}$$

If $c+k$ is sufficiently large, τ^m becomes sufficiently large, so that

$$\frac{d\left(\frac{\tau^m}{1-\alpha-(2-\alpha)f_\tau}\right)}{d\tau^m}=\frac{1-\alpha-(2-\alpha)f_\tau[1+\delta\tau^m]}{(1-\alpha-(2-\alpha)f_\tau)^2}>0.$$

Because τ^m decreases in θ, this means (EC.100) is larger when θ is larger. Therefore, $-I'(k)$ is larger when θ is larger. In this case, the convexity of $I(k)$ indicates the k is smaller, thus the monopolist spends more to reduce end-of-life cost.

For the fractional-revenue case, the optimal amount of investment is determined by

$$-I'(k)=\frac{e^{-\delta\tau^m}}{1-e^{-\delta\tau^m}}-\frac{dL}{d\tau^m}\frac{d\tau^m}{dk}.$$

where again the only difference between the definition of L and that of the fraction-profit case is that c instead of θc is used here. Following the same steps for proving Proposition 7 (from Equation EC.41 to Equation EC.43, page 45-46), the right-hand side becomes

$$\frac{e^{-\delta\tau^m}}{1-e^{-\delta\tau^m}}-\frac{d\phi}{d\tau^m}\frac{d\tau^m}{dk}=\frac{e^{-\delta\tau^m}}{1-e^{-\delta\tau^m}}\left[1-\frac{qf_\tau}{qf_\tau+q_\tau f\left(1-\frac{2-\alpha}{1-\alpha}f_\tau\right)}\right]$$

which is the same as (EC.100) except that θ does not appear as a multiplier. Applying the same argument, the monopolist spends more when θ is larger. ■

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