Risk-Aware Network Profit Management in a Two-Tier Market *

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We develop an optimization framework for the network carrier to manage profit in a two-tier market environment, a retail market where bandwidth is provisioned to serve uncertain demand and a wholesale market where bandwidth is bought and sold as a commodity. Our model is built upon mean-risk analysis. We discuss the selection of a risk index that is consistent with a stochastic efficiency criteria. We conduct numerical studies to investigate the influence of the network size and the carrier's risk averseness on bandwidth management, and profit implications of the bandwidth wholesale market.

1. Introduction

This paper gives a framework for carriers' network profit management. Models and techniques for macro-level bandwidth management within this framework are developed and presented here.

Uncertainty in (traffic) demand underlies the need for careful network profit management. In our models we use probabilistic distributions of demands as inputs. Obtaining such distributional information is in itself a major effort. However, forecasting and estimating traffic demand have been extensively studied, and much is known about relevant techniques [3], [5], [14].

Uncertainty breeds risk. A carrier must concern itself not only with mean profit and the strategies for its maximization, but also with the risk of profit falling below acceptable levels. Calculating risk and managing it are important components of network profit management, and also of this work. The optimization model in this paper incorporates a risk index. The rationale for its selection draws extensively from the mean-risk analysis that was originally developed in the finance community to address the needs of balancing growth and risk in portfolio management [6], [9]. Robust engineering extends the mean-risk analysis to areas like power capacity expansion, aircraft scheduling and structural design [11]. Risk in networking has its own characteristics. Risk is highly dependent on network scale, which is a theme that we develop further in our numerical studies. At the most basic level, the heterogeneity of demand between node pairs and capacity of links contributes to the variability of risk. Also, risk varies with service type, and, quite logically, this fact is reflected in market structure. This is exemplified by the two-tier market structure, which parallels a pair of contrasting services that is modeled here. Also note that carriers have varying attitudes towards risk, some being more averse to risk than others.

Considerable attention is given in this paper to find an appropriate measure of risk to use in the mean-risk approach. There are several candidates for such a risk index, which is incorporated in the objective function in the profit maximization. Some of these candidates, such as the variance of profit, lead to inferior solutions that are stochastically dominated by other feasible solutions. On the other hand, there are candidates, such as the Tail Value at Risk (TVaR), which are

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stochastically efficient but rather difficult to handle analytically and in the optimization. We show that the standard deviation of network profit as a measure of risk is a good compromise.

Bandwidth management is of vital importance to risk-aware profit optimization. In this paper, bandwidth sharing and management is at macro-level, and rather different from that considered at the connection or packet levels. Stochastic traffic engineering [12] and the emerging technology of MultiProtocol Label Switching(MPLS) are primary enablers. The optimization in our model is with respect to the topology of the paths serving end-to-end demands and the amount of provisioned bandwidth on these paths, among other decision variables.

The two-tier market structure is an important aspect of our model. It is also an important mechanism for risk management. The first tier is a retail market where bandwidth is sold as a service. The second tier is a wholesale market where bandwidth is bought and sold as a commodity. The buying and selling of bandwidth is an important facility for bandwidth and profit management. Demand is deterministic in the wholesale market, so that there is no risk in profit management, but the price for carrying each unit of bandwidth is low. In the retail market the carrier can charge a premium price, but the demand is stochastic, so that there exists the risk of profit shortfall. By proper dimensioning of bandwidth provisioned for each market, the carrier can maximize mean profit at its acceptable risk level. In our modeling framework the existence of the wholesale market, which is not widely established in today's marketplace, may be readily removed, in which case the stochastic demand in the retail market becomes the main focus. One of the goals of our work is to provide a platform for quantifying the cost-benefit tradeoffs that the wholesale market brings to profit management in networking.

In a recent paper [12], we modeled a wholesale market in which the carrier is allowed to sell bandwidth. This paper extends the model by allowing the carrier to buy bandwidth as well. A central theme of [12] is the effect of demand uncertainty on network traffic engineering. In this paper the focus is different, with network profit management the dominant theme. Also new are the choice of risk index, the analysis of stochastic efficiency and the trade-off between the latter and tractability. The numerical studies of the implications of variations of carriers' risk averseness is new here. Also new are studies of the efficient frontier and the benefits of the wholesale market to carriers operating networks with diverse capacity levels.

Our framework has the potential of providing the basis for studying key future developments in the networking industry. For instance, if the industry structure stabilizes to one with several competing carriers with distinct and possibly overlapping footprints, then resource sharing among all or a subset of these carriers will benefit not only the participating carriers but society and consumers also. Modeling frameworks, such as the one here, will be needed to quantify the value proposition from such bandwidth sharing agreements. In the present model the wholesale market with which the carrier interacts may be viewed as a proxy for the combination of all the other participating carriers. Clearly this view leads to dynamic, game-theoretic iterations composed of actions and reactions of the players, in which the basic moves by each player may be computed by an extension of the present model.

The paper is organized as follows. In Section 2, we present a general formulation of the problem. The modeling of the risk factor in the objective function is discussed in detail in Section 3. In Section 4, we develop several numerical examples to illustrate some implications of our model. Conclusions are given in Section 5.

2. Problem Formulation

2.1. Two-Tier Market

Our framework features a two-tier market structure. The first tier is a retail market where bandwidth is sold as services, such as voice, data, and video. Both the user base of the carrier and the usage rate of its customers are subject to random fluctuation, introducing uncertainties in the bandwidth demand and retail revenue. We reflect such uncertainties in our framework by letting retail demands be random variables.

The second tier is a wholesale market where bandwidth is sold as standardized commodity, e.g., DS3 circuits. In our framework the market is an alternative source of revenue to the carrier where it can sell bandwidth wholesale. The market also provides a short-term supply of capacity to the carrier where it can buy bandwidth to augment installed capacity in its network to serve retail demand. In this sense, the wholesale market provides a mechanism for many carriers to share resources and revenue, like the ones proposed in [1] and [8], and can be supported by an appropriate engineering architecture [2].

We assume that the carrier under consideration is one of many participants in the wholesale market. Hence no carrier can dominate total demand and supply, and thereby significantly influence bandwidth price. This implies that the carrier's revenue from wholesale and cost of buying bandwidth follow deterministically from its buying and selling decisions.

2.2. Admissible Route Sets

QoS and policy considerations are major constraints on provisioning decisions. The notion of *admissible route sets* allows these constraints to be taken into account in the optimization. Let $\mathcal{R}(v)$ denote the set of admissible routes for origin-destination pair v. Different admissible route sets for wholesale and retail services between the same node pair are allowed. For example, routes may be required to have lengths not exceeding specified thresholds, on account of propagation delay, and there may also be restrictions on the number of hops, since each hop is associated with a switching node and consequent incremental delay. The admissibility of a route may also depend on policy, which might reflect diverse considerations, such as security, the capability of switching nodes in the routes to handle certain services, etc. Generating the admissible route sets is a substantial task in itself. We assume that these sets are given in this paper.

2.3. Model Formulation

Define the *extended network* as a collection of resources that is accessible to the carrier. An extended network is represented by a graph $\mathcal{G} = (\mathcal{N}, \mathcal{L})$, where $\mathcal{N}$ and $\mathcal{L}$ are node and link sets, respectively. $\mathcal{L}$ is the union of $\mathcal{L}_U$ and $\mathcal{L}_V$, where $\mathcal{L}_U$ is the set of installed links on which the carrier owns deployed capacity, and $\mathcal{L}_V$ is the set of virtual links on which the carrier has the option to buy capacity. $\mathcal{L}_U$ and $\mathcal{L}_V$ are not necessarily mutually exclusive. The capacity that the carrier owns on link l, C_l, is an input parameter. The amount of capacity that the carrier buys on link l, b_l, is a decision variable. Then $C_l = 0$ for $l \in \mathcal{L} - \mathcal{L}_U$, and $b_l = 0$ for $l \in \mathcal{L} - \mathcal{L}_V$. Assume that the market price for buying capacity on link $l \in \mathcal{L}_V$ is p_l per unit of bandwidth. Then $\sum_{l \in \mathcal{L}_V} p_l b_l$ is the carrier's cost of buying bandwidth, and $C_l + b_l$ is the amount of bandwidth on link l at the carrier's disposal.

Let $\mathcal{V} = \{(v_i, v_j) : v_i \in \mathcal{N}, v_j \in \mathcal{N}\}$ be the set of all node pairs. $\mathcal{V}_1 \subset \mathcal{V}$ is the collection of node pairs between which there are retail demands, and $\mathcal{V}_2 \subset \mathcal{V}$ is the collection of node pairs between which wholesale of bandwidth is feasible.

Retail demand between $v \in \mathcal{V}_1$ is characterized by a random variable, with its Probability Density Function (PDF) denoted by $f_v(x)$, and Cumulative Distribution Function (CDF) denoted by $F_v(x)$. Let $d_v (v \in \mathcal{V}_1)$ be the amount of capacity provisioned to serve retail demand between v. The provisioned quantity, d_v, can be routed on one or more admissible routes. Denote the admissible route set for $v \in \mathcal{V}_1$ by $\mathcal{R}_1(v)$ and let $\xi_r (r \in \mathcal{R}_1(v))$ be the amount of capacity provisioned on route r. Then $d_v = \sum_{r \in \mathcal{R}_1(v)} \xi_r$. We require that $0 < \bar{d}_v \leq d_v$ for all $v \in \mathcal{V}_1$, where $\bar{d}_v$ is defined as the minimum amount of bandwidth that must be provisioned for retail demand between v to satisfy the grade of service (GOS) that the carrier offers to its retail customers. Denote by T_v the random retail demand between node pair v. Then $t_v(d_v) = \min(T_v, d_v)$ is the amount of retail demand that is carried between v. Note that the revenue earned by the carrier is based on the carried demand. Let $m_v(d_v)$ and $s_v^2(d_v)$ be the mean and variance of t_v,

$$\begin{aligned} m_v(d_v) &= \int_0^{d_v} x f_v(x)dx + d_v \bar{F}_v(d_v) = \int_0^{d_v} \bar{F}_v(x)dx \\ s_v^2(d_v) &= \int_0^{d_v} x^2 f_v(x)dx + d_v^2 \bar{F}_v(d_v) - m_v^2(d_v) = 2\int_0^{d_v} x\bar{F}_v(x)dx - m_v^2(d_v), \end{aligned} \tag{1}$$

where $\bar{F}_v(x) \equiv 1 - F_v(x)$. Note that

$$\frac{\partial m_v(d_v)}{\partial d_v} = \bar{F}_v(d_v) \geq 0, \text{ and } \frac{\partial s_v^2(d_v)}{\partial d_v} = 2(d_v - m_v(d_v))\bar{F}_v(d_v) \geq 0. \tag{2}$$

Let π_v be the revenue earned for each unit of retail traffic carried between v. The total revenue derived from serving retail demand between v is a random variable $\pi_v t_v(d_v)$, for which the mean is $\pi_v m_v(d_v)$ and the variance is $\pi_v^2 s_v^2(d_v)$.

Similarly, let y_v be the amount of bandwidth provisioned for wholesale between $v \in \mathcal{V}_2$, $y_v = \sum_{r \in \mathcal{R}_2(v)} \phi_r$, where $\mathcal{R}_2(v)$ is the admissible route set for $v \in \mathcal{V}_2$ and ϕ_r is the amount of provisioned bandwidth on route r to carry wholesale traffic. Suppose e_v is the unit wholesale price between node pair v. Then the wholesale revenue is $e_v y_v$.

The carrier's profit equals combined revenue from serving retail demand and selling capacity, minus the cost of buying bandwidth,

$$W = \sum_{v \in \mathcal{V}_1} \pi_v t_v + \sum_{v \in \mathcal{V}_2} e_v y_v - \sum_{l \in \mathcal{L}_V} p_l b_l. \tag{3}$$

Due to random retail demand, $t_v (v \in \mathcal{V}_1)$ are random variables, and consequently so is W. The carrier's objective, denoted by Θ, is a function of W. The formulation of Θ should reflect the carrier's desire to increase mean profit and reduce the risk of profit shortfall due to the randomness of W.

To summarize, the above discussion leads to the following optimization model:

$$\max \Theta(W(d_v, y_v, \xi_r, \phi_r, b_l)) \tag{4}$$

subject to:

$$\sum_{r \in \mathcal{R}_1(v)} \xi_r = d_v \ (v \in \mathcal{V}_1), \ \sum_{r \in \mathcal{R}_2(v)} \phi_r = y_v \ (v \in \mathcal{V}_2), \tag{5}$$

$$\sum_{r \in \mathcal{R}_1(v): l \in r} \xi_r + \sum_{r \in \mathcal{R}_2(v): l \in r} \phi_r \leq C_l + b_l \ (l \in \mathcal{L}), \tag{6}$$

$$0 < \bar{d}_v \leq d_v \ (v \in \mathcal{V}_1), \quad 0 \leq y_v, \xi_r, \phi_r, b_l, \ (v \in \mathcal{V}_2, r \in \mathcal{R}_1(v) \text{ or } \mathcal{R}_2(v), \ l \in \mathcal{L}_V). \tag{7}$$

We discuss the objective function in the next section. As for constraints, (5) specifies bandwidth provisioning for both retail and wholesale demands on admissible routes; (6) limits the total amount of provisioned bandwidth from exceeding the sum of installed and purchased link capacities. In (7), $\bar{d}_v$ is defined as the minimum bandwidth that must be provisioned for retail demand between node pair v. The value of $\bar{d}_v$ is determined by the grade of service that the carrier offers to its retail customers. The equation also constrains all decision variables to be non-negative.

3. Modeling Risk

In this section, we take uncertainty in demand into consideration in the development of an objective function that features both the maximization of mean profit and the containment of the risk of profit shortfall. We review two relevant risk modeling frameworks in Section 3.1 and discuss our characterization of profit risk in Section 3.2.

3.1. Relevant Frameworks for Risk Modeling

3.1.1. Mean-Risk Model

Mean-risk analysis addresses the issue of risk averseness by offering a broader optimization objective. The approach starts by developing a risk index, which is a quantitative measure of the risk of profit shortfall, based on the profit distribution. It then maximizes the weighted combination of the mean profit and the risk index, i.e., $mean - \delta * (risk\ index)$, where $\delta \geq 0$ is a parameter. Different levels of risk averseness can be reflected by choosing different values for δ. A higher value of δ indicates greater willingness to sacrifice the mean profit to avoid risk.

3.1.2. Stochastic Dominance

Stochastic dominance theory defines a partial ordering of random variables based on their probability distributions [7]. Let W_1 and W_2 be two random variables, which represent profits under two different bandwidth management decisions. Then W_1 stochastically dominates W_2 to the first degree iff the former renders the carrier a better chance to exceed *any* profit target w, i.e.,

$$\forall w, \ Pr(W_1 \geq w) \geq Pr(W_2 \geq w). \tag{8}$$

Furthermore, W_1 stochastically dominates W_2 to the second degree iff

$$\int_w^\infty Pr(W_1 \geq \zeta)d\zeta \geq \int_w^\infty Pr(W_2 \geq \zeta)d\zeta \quad \forall w. \tag{9}$$

We consider a solution to our problem to be stochastically efficient if the corresponding profit distribution is not dominated in either degree. Note that it suffices to prove efficiency by showing that the profit distribution is not subject to second-degree dominance, which, by definition, is a necessary condition for dominance in the first degree.

3.2. Formulation of the Objective Function

We formulate the objective function as an instance of the mean-risk model with a constraint that the optimal solution is stochastically efficient. Whether the latter constraint can be satisfied depends on the characteristics of the profit distribution and the choice of the risk index. A common approach is to use variance as the risk index. If the profit is normally distributed, a solution that maximizes (mean - $\delta *$variance) is provably stochastically efficient [7]. For other distributions, the optimal solution to the mean-variance model can be stochastically dominated, which, for our problem, will be shown in Section 3.2.1. On the other hand, applying other risk measures that guarantee stochastic efficiency, such as Tail Value at Risk defined in Section 3.2.2, leads to models that are too complex to solve. As a compromise, in Section 3.2.3 we propose standard deviation as the risk measure.

3.2.1. Variance

Variance is a natural candidate for the risk index since it has been widely used in many mean-risk models. The objective function that uses the variance as the risk index is

$$\Theta = E(W) - \delta Var(W) = \sum_{v \in \mathcal{V}_1} \pi_v [m_v(d_v) - \delta \pi_v s_v^2(d_v)] + \sum_{v \in \mathcal{V}_2} e_v y_v - \sum_{l \in L} p_l b_l. \tag{10}$$

Notice that (10) reflects the aforementioned assumption that demands between different node pairs are independent. Applying (1),

$$\frac{\partial \Theta}{\partial d_v} = \bar{F}_v(d_v)[1 - 2\delta\pi_v(d_v - m_v)] = \bar{F}_v(d_v)[1 - 2\delta\pi_v \int_0^{d_v} F_v(x)dx]. \tag{11}$$

The equation $2\delta\pi_v \int_0^{d_v} F_v(x) = 1$ has an unique solution, $\hat{d}_v$, such that

$$\frac{\partial \Theta}{\partial d_v} \geq 0, \ \frac{\partial^2 \Theta}{\partial d_v^2} \leq 0 \text{ if } d_v \leq \hat{d}_v \text{ and } \frac{\partial \Theta}{\partial d_v} < 0, \text{ if } d_v > \hat{d}_v. \tag{12}$$

Based on this observation, we impose $\max(\bar{d}_v, \hat{d}_v)$ as an upper bound on d_v, which makes our model a concave maximization problem that can be solved efficiently. If $\hat{d}_v \leq \bar{d}_v$, then $d_v = \bar{d}_v$ by (7). Otherwise, $d_v \leq \hat{d}_v$, which is a new constraint that reduces the original feasible region (a polyhedron defined by (5), (6), and (7)) to a convex set on which the objective function (10) is concave. This additional restriction on d_v does not affect the optimal solution since by (12), increasing d_v beyond $\hat{d}_v$ only reduces the value of Θ.

The above analysis implies that the optimal solution of the mean-variance model can be stochastically dominated. Since the optimal value of d_v is bounded by $\max(\bar{d}_v, \hat{d}_v)$, when the network has more than enough installed capacity to serve every retail demand to its upper-bound and wholesale is limited, some bandwidth will be left idle in the optimal solution. In this case, one can generate a new feasible solution by increasing the bandwidth provisioned to node pairs that are connected by underused links while keeping everything else unchanged. The new solution always has a better chance to get more profit than the optimal one and thus stochastically dominates the latter to the first degree.

3.2.2. Tail Value at Risk

Besides variance, other distributional parameters have also been proposed as candidates for the risk index. Of particular interests are those that guarantee stochastic efficiency of the optimal solution. One example is the Tail Value at Risk (TVaR), defined as:

$$TVaR(p) = \int_0^p q_W(\eta)d\eta, \tag{13}$$

where $q_W(p) = \inf\{w | Pr(W \leq w) \geq p\}$ is the p-quantile of profit W [7], [13].

We do not use TVaR as the risk index in our model because it cannot be formulated as an explicit function of decision variables, which makes optimization computationally difficult.

3.2.3. Standard Deviation as Risk Index

Our choice of risk index is standard deviation of profit, $s(W)$, which is computationally more tractable for optimization than TVaR. For networks with few node-pair demands, an optimal solution to the mean-standard-deviation model may still be stochastically dominated. However, the model is asymptotically compatible with the efficiency criteria as the number of node-pair demands increases. This property serves our purpose well since bandwidth transport networks usually have tens or even hundreds of nodes, and thus a large number of node pairs.

As in (3), the random component of profit W is a summation of many *independent* random retail revenues, $\pi_v t_v(d_v)$. From (2) the variance of each revenue, $\pi_v^2 s_v^2(d_v)$, monotonically increases with d_v, and thus has a lower bound $\pi_v^2 s_v^2(\bar{d}_v)$ from (7), and an upper bound $\pi_v^2 s_v^2(+\infty) = \pi_v^2 var(T_v)$. Therefore, it is generally true that for every feasible solution

$$\lim_{|\mathcal{V}_1|\to+\infty} \frac{\pi_{v'}^2 s_{v'}^2(d_v)}{\sum_{v\in\mathcal{V}_1} \pi_v^2 s_v^2(d_v)} = 0, \ \forall v' \in \mathcal{V}_1,$$

i.e., Lindeberg's condition is satisfied so the Central Limit Theorem can be applied as follows,

$$\frac{W - E(W)}{s(W)} = \frac{\sum_{v\in\mathcal{V}_1} \pi_v[t_v(d_v) - m_v(d_v)]}{\sqrt{\sum_{v\in\mathcal{V}_1} \pi_v^2 s_v^2}} \to N(0,1), \text{ as } |\mathcal{V}_1| \to +\infty. \tag{14}$$

where $N(0,1)$ is standard normal distribution. It follows that if $|\mathcal{V}_1|$ is sufficiently large,

$$\begin{aligned} TVaR(p) &= \int_0^p q_W(\eta)d\eta = \int_{-\infty}^{q_W(p)} x dF_W(x) \\ &\approx \int_{-\infty}^{q_N(p)} [E(W) + s(W)\eta]dF_N(\eta) = p[E(W) - \frac{e^{-q_N^2(p)}}{p\sqrt{2\pi}} s(W)], \end{aligned} \tag{15}$$

where $q_N(p)$ is the p-quantile and $F_N(\eta)$ is the CDF of $N(0,1)$. Since $e^{-q_N^2(p)/2}/(p\sqrt{2\pi})$ changes monotonically from 0 to $+\infty$ as p varies from 1 to 0, $\delta = e^{-q_N^2(p)/2}/(p\sqrt{2\pi})$ has an unique inverse $p(\delta)$. By (15), the solution that maximizes $E(W) - \delta s(W)$ also maximizes $TVaR(p)$ for $p(\delta)$, and thus is stochastically efficient.

Using the standard deviation as the risk index, the objective function in (4) becomes

$$\max \Theta(W(d_v, y_v, \xi_r, \phi_r, b_l)) = \sum_{v\in\mathcal{V}_1} \pi_v m_v(d_v) + \sum_{v\in\mathcal{V}_2} e_v y_v - \sum_{l\in\mathcal{L}_V} p_l b_l - \delta \sqrt{\sum_{v\in\mathcal{V}_1} \pi_v^2 s_v^2(d_v)}. \tag{16}$$

Though more complicated than the aforementioned use of variance, the optimization of (16) is still tractable, as shown in [12].

4. Numerical Studies

In this section, we discuss the implications of our model through numerical examples. We first describe the network topology and base case scenario in Section 4.1. In Section 4.2, we analyze the influence of the carrier's risk averseness on the optimization results. In Section 4.3, we examine the impact of the bandwidth wholesale market. In Section 4.4, we compare differences in optimal bandwidth management for networks of different sizes.

4.1. Framework and Base Case

We consider a sample network that has 12 nodes and 14 installed and virtual links. The network topology is shown in Figure 1, where link 4, represented by a dash line, is a "pure" virtual link that has no installed capacity. We assume that retail demands are symmetric in both directions and characterized by the Truncated Gaussian distribution, with support on the nonnegative real line and the following distribution function

$$Pr(T_v \le x) \equiv F_v(x) = \frac{1}{\sqrt{2\pi}\sigma_v G_v} \int_0^x e^{-(\omega-\mu_v)^2/2\sigma_v^2} d\omega, \quad x \ge 0, \tag{17}$$

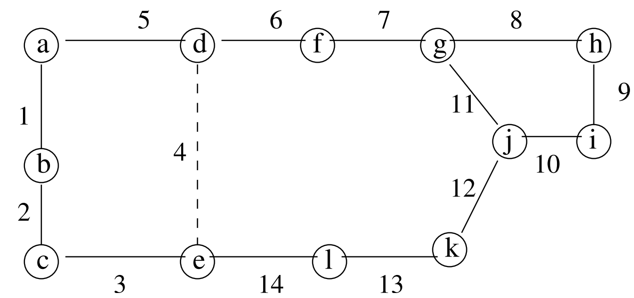

Figure 1. Network Topology

where the normalizing parameter is $G_v = Erfc(-\mu_v/(\sqrt{2}\sigma_v))/2$.

We assume $\sigma_v = \kappa\mu_v$ and let κ be the same for all v. For small value of κ (e.g., $\kappa \leq 1/3$) and positive μ_v, $G_v \approx 1$, in which case the truncation effect can be ignored and μ_v and σ_v are close approximations to the mean and standard deviation of the demand. In this case, κ is the coefficient of variation. We assume for all $v \in \mathcal{V}_1$, $\mu_v = \bar{\mu}$. Let h_v be the minimum number of hops between node pair $v \in \mathcal{V}_1$. Then $\sum_{v \in \mathcal{V}_1} \mu_v h_v = \bar{\mu} \sum_{v \in \mathcal{V}_1} h_v$. We define the ratio of this quantity to the total installed network capacity to be the network *load*, denoted by ρ, i.e., $\rho = (\bar{\mu} \sum_{v \in \mathcal{V}_1} h_v)/(\sum_{l \in \mathcal{L}} C_l)$.

In our base case, all links have installed capacity 200 except link 4 which has zero installed capacity. Retail demand exists between every node pair of the network. Bandwidth buying is allowed on every installed and virtual link, and wholesale is allowed between node pairs that are directly connected by these links. For demand distributions, $\rho = 0.7$ and $\kappa = 0.35$, which implies $\mu_v = 10.34$ and $\sigma_v = 3.62$ for all $v \in \mathcal{V}_1$. The unit retail price $\pi_v = 50h_v$, where h_v is the minimum number of hops between node pair v. The unit wholesale price $e_v = 0.2\pi_v$. The unit price for buying bandwidth $b_l = 1.05e_{v(l)}$, where l is the direct link between v_l [2]. A path between node pair v is admissible for retail demand iff the number of links on this path does not exceed the minimum number of hops plus two, i.e., $h_v + 2$. To satisfy GoS requirement, the amount of capacity provisioned to serve each retail demand should at least equal to the mean demand, i.e., $\bar{d}_v = E(T_v)$. Unless otherwise noted, the risk parameter $\delta = 0.5$.

4.2. Efficient Frontier

Table 1 shows the influence of the carrier's risk averseness on key model outputs where δ ranges from a conservative value of 2.5 to the most aggressive value of 0 (in which case the carrier maximizes the expected profit only) in steps of 0.5. The carrier buys more bandwidth when it becomes less concerned about minimizing risk, as indicated by 36% jump (from 2656.4 to 3618.3) of the total buying expense in column 2. Allocation of total bandwidth to different markets is shown in column 3. In the most conservative case, 24.7% of total capacity is devoted to wholesale, which earns a lower but guaranteed unit revenue. This percentage is reduced to 20.3% in the most aggressive case. Consequently, the carrier's wholesale revenue drops 15% from 7034.3 to 5983.9 (column 4). This change makes the expected revenue from retail a more significant portion of total expected revenue, as shown in columns 5 and 6. The cumulative profit impact of these changes is summarized by the efficient frontier in Figure 2. Each point gives an optimized combination of mean and standard deviation of profit at a given δ value, representing the maximum expected profit (reward) obtainable at a given level of risk, or the

[2] Letting the buying price to be slightly higher than the selling price avoids an unreasonable situation that it costs nothing for one to buy and sell bandwidth between the same node pair. The price difference can be interpreted as the transaction cost borne by the buyer.

Table 1
Changes of Optimal Decisions with Risk Parameter

δ	total buying expense	% wholesale bandwidth	wholesale revenue	expected retail revenue	% retail revenue
2.5	2656.4	24.7%	7034.3	84995.3	92.4%
2.0	2833.1	23.6%	6780.2	85638.5	92.7%
1.5	3020.9	22.7%	6566.1	86189.6	92.9%
1.0	3228.6	21.9%	6363.9	86703.8	93.2%
0.5	3428.0	21.1%	6169.4	87155.3	93.4%
0	3618.3	20.3%	5983.9	87548.7	93.6%

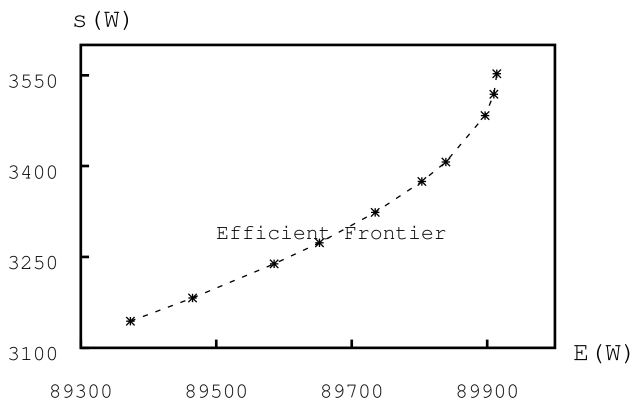

Figure 2. Efficient Frontier

minimum risk the carrier has to take for a given reward.

4.3. The Value of the Wholesale Market

In the following, we discuss the benefit of having a wholesale market. Define

$$\Delta(E_W) = \frac{E(\tilde{W}) - E(\bar{W})}{E(\bar{W})}, \ \Delta(s_W) = \frac{s(\tilde{W}) - s(\bar{W})}{s(\bar{W})},$$

where $\tilde{W}$ and $\bar{W}$ are optimized profit with and without the wholesale market, respectively. In Figure 3a, $\Delta(E_w) > 0$, which implies that the wholesale always benefits the carrier. The figure shows how $\Delta(E_W)$ varies with the wholesale price (given as a fraction of the retail revenue). When the price is low, the carrier takes advantage of the wholesale market by buying relatively cheap bandwidth to serve more lucrative retail demand. In this region, an increase in price reduces improvement of expected profit, as reflected by decreasing $\Delta(E_W)$. The price increase eventually makes it profitable to sell more and buy less bandwidth in the wholesale market. At this point, $\Delta(E_W)$ starts to increase with the price. The location of the switching point depends on the network load. In a lightly loaded network ($\rho = 0.6$), $\Delta(E_W)$ starts increasing at a low price level (5% of unit revenue of retail demand). In a more heavily loaded network ($\rho = 0.8$), $\Delta(E_W)$ does not start to increase until the price reaches a relatively high level.

In Figure 3b, $\Delta(s_W)$ is positive when wholesale price is low, indicating profit variability increases as a result of buying bandwidth to serve more retail demand. As price increases,

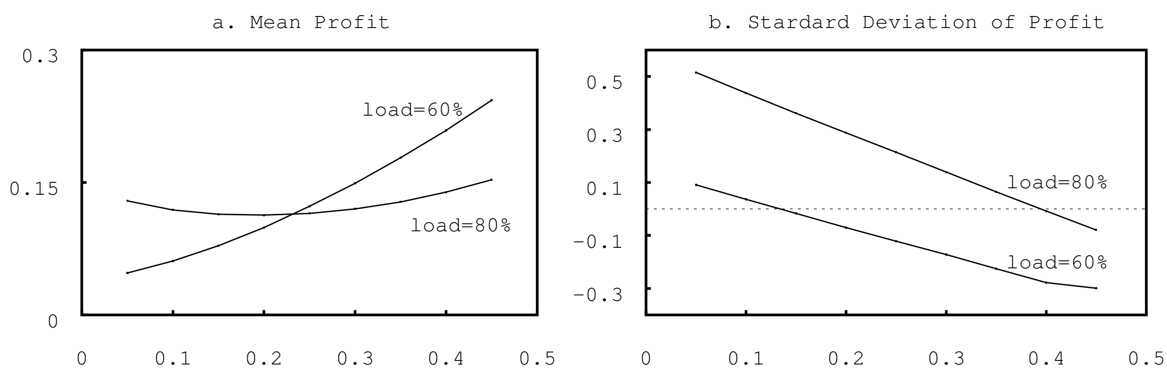

Figure 3. Percentage Change of Profit Distribution (x: e_v/π_v, y: $\Delta(E_W)$ or $\Delta(s_W)$)

bandwidth selling becomes more profitable, so less capacity is provisioned to retail demand and variation in profit is reduced. Consequently, $\Delta(s_W)$ decreases and eventually becomes negative.

4.4. Influence of Network Scale

We now discuss differences in optimal bandwidth management for networks of various sizes. We start with the base case and scale installed capacity by a constant (γ) uniformly across all links. By keeping the load factor ρ unchanged, we scale the mean retail demand by the same factor. As the average volume grows, the variation of retail demands may or may not decrease. Similar to the "power law" in [3], we scale the standard deviation by γ^a, where $a \geq 0$. It is easy to verify that when $a = 1$, optimal values of all decision variables and the objective function change proportionately to γ. In the following example, we set $a = 0.5$ to model the situation of decreasing uncertainty to network scale.

We consider six differently sized networks ($\gamma = 0.5, 0.75, 1, 1.5, 2, 3$) and vary wholesale price as different fractions of retail revenue. We compare the carrier's buying and selling activities in Table 2, where the normalized balance of wholesale bandwidth, defined as $(\sum_{v \in \mathcal{V}_2} y_v - \sum_{l \in \mathcal{L}_V} b_l)/\sum_{l \in \mathcal{L}_U} C_l$, is reported.

When the wholesale price is low, smaller carriers have negative balances, indicating that they are net buyers in the wholesale market. Larger carriers are net sellers who have positive balances. At the low price level, it does not cost much to buy enough bandwidth to accommodate the most optimistic demand scenario. Smaller carriers buy even more because higher uncertainties in their retail demand imply a better chance of a large upward demand swing. As the price increases, smaller carriers continue to buy more and sell less bandwidth than large carriers, but the difference diminishes. When the wholesale price is high, carriers of all sizes are better-off to provision capacity to retail demands to the level that is just enough to satisfy GoS, and wholesale the rest. In our example, the minimum quantity to satisfy GoS is set at mean demand, which is proportional to the network size. The assumption makes the proportion of bandwidth provisioned to retail demand more or less equal across networks of different sizes, which implies that their normalized wholesale balances become uniform.

5. Conclusion

We have developed an optimization model to support bandwidth management decision-making in a two-tier market environment. Our model is based on the mean-risk framework with the requirement that the optimal solution is stochastically efficient. We have discussed the tradeoffs

Table 2
Normalized Wholesale Bandwidth Balance

	Network Scale					
e_v/π_v	$\gamma = 0.5$	$\gamma = 0.75$	$\gamma = 1.0$	$\gamma = 1.5$	$\gamma = 2.0$	$\gamma = 3.0$
0.10	-12.13%	-4.18%	0.45%	5.80%	9.05%	12.89%
0.30	13.92%	17.12%	18.93%	20.90%	22.12%	23.57%
0.50	28.16%	29.47%	29.83%	29.98%	30.00%	30.00%

in the selection of the risk index and have proposed the use of standard deviation of total profit. We have analyzed the impacts of carrier's risk averseness on bandwidth management and discussed benefits of the wholesale market. Generalizing our model to study interactions of many carriers in a wholesale market is an interesting future extension.

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